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PRACTICAL GEOMETRY.

MANUAL OF PRACTICAL PHYSICS

PART I.—PRACTICAL GEOMETRY

BY

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FOURTH EDITION

INDIAN PRESS: ALLAHABAD.

1905.

 [Price seven annas.

ALLAHABAD
PRINTED AT THE INDIAN PRESS

PREFACE.

Some five years ago Elementary Science was introduced into the Upper Classes of the Anglo-Vernacular Schools of these Provinces, as one of the optional subjects in the School Final Course. At first only a few of the better schools were allowed to teach the subject; as facilities extended, other schools were recognized; now there are only a few schools where the subject is not taught.

It is a popular subject and probably has some educational value, although there can be no doubt its educational value could be very largely increased. The courses in use in the schools have several important defects. They are too wide and shallow, too many subjects are included and none are studied thoroughly. The text-book has become too prominent, facts are sought from the printed page, and Nature is little studied. The subject has become to a large extent a mere 'Cram' subject, in which there is very little scientific training, and the pupil who correctly reproduces the text-book is supposed to know some "Science". In the best case the course merely gives additional exercise to a faculty already overburdened, and in a large number of cases it is, to be feared, does almost as much harm as good.

It must be confessed, however, that till comparatively recently, Elementary Science Teaching in even

good schools in England was in very much the same condition. Only within the last few years have the possibilities and advantages of a thorough study of science been generally recognised. But now the old loose methods of study have been abandoned in most schools and a fairly successful attempt made to develop Science Teaching on more rational lines.

Among many points of difference between the old and new methods of teaching the subject, the most striking is the THOROUGHNESS of the new as contrasted with the vagueness and looseness of the old.

This greater thoroughness appears in various ways—

(1) The range of subjects has been greatly reduced. It is now recognised that no beginner can with profit study even simple facts bearing on eight subjects. (In Elementary Primers the field covered was in some cases truly alarming. A little Statics, a little Dynamics, a little Hydrostatics, a little Heat, a little Optics, a little Acoustics, a little Electricity, a little Magnetism, and a little Chemistry—all within the covers of a tiny book.)

(2) Experimental work is insisted on. Although it may be said that experiments formed a part of the old method, it is true only in a very limited sense. The experiments were conducted by the teacher, the boys looking on. Now the boys themselves carry out the experiments and learn their facts at first hand, and to much better purpose.

(3) Exact measurements are now required, and phenomena are studied quantitatively as well as qualitatively. A fact is now not merely observed, it is also measured, and its precise relation to other facts determined. The old school, for example, were satisfied with shewing that air is compressible, the new school seek to help the student to discover the precise amount of extra pressure required to produce a particular amount of compression, and from his observations to deduce a generalized statement or Physical Law. Not only is the student's store of facts thus greatly increased, his knowledge of them becomes much more effective and intimate. The strain on the memory is at the same time diminished, while other faculties are brought into use in the search for specific relations between related phenomena.

It will thus be seen that this greater thoroughness has been accompanied by radical changes in methods of teaching; the same facts are studied but now in a different way, and with much better adaptation of means to ends. On the old system much of the work was vague and indefinite, now all is precise and exact. A study of the methods employed to secure this precision and exactness is alone the study of Science and a training in these methods is a Scientific Training.

A Scientific Training of this nature is, however, very different from the study of science as hitherto prosecuted in our schools, and the question may well

arise, Is it possible or desirable to begin such a training in schools, can it be profitably commenced before the student enters College?

Some may doubt the desirability of this, in view of the long list of "subjects" that now confront the bewildered school boy, and for this opinion there is, of course, a good deal to be said. Even the cleverest boys must leave some subjects on one side; no one can pursue with advantage all possible branches of study; something must be sacrificed.

But while this is so, there must always be some who have a special liking for scientific work, and who would naturally take up a science subject if it were desirable and possible to introduce such a subject into school courses. Can, therefore, the scientific training of these boys be profitably begun in school? Many eminent English Educationalists think it can. For what constitutes a scientific training? In such a training, hand and eye are trained, habits of neatness and accuracy acquired, clear and orderly methods of recording facts learned, exact measurements made, and the powers of observation quickened. Now if we look at this fairly, we must see that such a training may be begun almost at infancy.

The training of hand and eye may be begun very early. Habits of neatness and accuracy are best acquired young. Clear and orderly methods of recording facts can be learned even by children. Chiefest of all, the naturnal inquisitiveness of children is the best aid a teacher can have in leading his pupils to the study of nature. Hence it is that an elementary training

in scientific work now occupies a very prominent place quite low down in many English Schools.

Indeed, the educational value of such a training has become so manifest that in quite a large number of schools a short elementary course has been made compulsory on all. This is quite intelligible. It must be an advantage to every one to have a certain amount of training. A good eye, a trained hand, a neat style, a habit of carefulness and accuracy, are valuable possessions in every line of life.

But if desirable, is it possible to introduce this training into our schools here? This question is hard to answer. The present course is an attempt to introduce such a training; whether it will succeed or not, remains to be seen. In India there are many difficulties in the way, the most prominent being perhaps, the lack of trained teachers. This difficulty is peculiarly felt at the commencement, when the course is new and strange, and when a competent sympathetic teacher would be invaluable. If, however, our teachers grasp the idea underlying such a course as this, and teach it in the proper spirit, it will succeed, but if they are careless, slipshod, or mechanical, the scheme is bound to fail.

I have endeavoured to lessen this difficulty in various ways—

(1) I have confined Part I of the course to the most elementary operations. I have sought a training to hand and eye by exercises in drawing, without mechanical aid, straight

lines of given length, angles of given size, and simple geometrical figures. I have introduced only the very simplest measurements,—measurements of lengths and angles. I have entirely excluded all, strictly *Physical* work, so much so that the title 'Manual of Practical Physics' almost seems a misnomer. I am convinced, however, that this is wise, for even the very simplest *physical* ideas are really difficult to the beginner,—much more so than geometrical work of even considerable complexity. The slow and late progress of physical ideas, compared with the early advance of mankind in geometrical work sufficiently illustrates this.

(2) I have tried to give the fullest and most explicit directions for conducting the work.

(3) I have drawn up a note book, to be used with the Manual, in which the proper method of proceeding is more fully indicated. In it, also, some hints are given regarding the best way of recording facts.

It is hoped that the Teacher will thus never be at a loss in guiding his pupils in the right track.

I have only to add that the Teacher should not use the books slavishly, or mechanically, but in such a way as to secure the best results. He should keep always in mind the objects aimed at in the course.

They are :—

1. To train hand and eye.
2. To develop habits of neatness and accuracy.
3. To teach clear and orderly methods of recording facts.
4. To give a training in methods of exact measurement.
5. To cultivate the powers of observation.

(1) Regarding the training of hand and eye: He may find it desirable to give more exercises for practice than I have given. He should, in this matter, use his own judgment and give extra exercises wherever his pupils may require it.

(2) The teacher should particularly insist on the utmost carefulness and neatness. Slipshod and careless work should be severely punished and good work suitably rewarded.

(3) It will probably be found useful to allot marks to each boy's work after every lesson, the note books to be collected by the Teacher after the lesson and taken home for this purpose. On no account should the boys be allowed to take their note books out of the class room—for obvious reasons,—they should receive them at the beginning of each lesson and hand them up again at the end.

To encourage the boys to try their best it would be well to offer prizes for the neatest and most accurate note book showing the greatest amount of work done. But in no case should quantity count for quality.

The following is an outline of this part of the course :—

CHAPTER I.—STRAIGHT LINES.

Lessons 1-6. Exercises for Hand and Eye.

„ 7-10. Methods of comparing the Lengths of Lines, with definition and description of a Unit of Length.

„ 11-13. Methods of measuring the lengths of lines correct to (*a*) centimetres, (*b*) millimetres, (*c*) tenths of millimetres.

Lesson 14. Errors.

CHAPTER II.—ANGLES.

Lessons 15-17. What constitutes an angle and how the sizes of angles are determined, and compared

„ 18-19. Right angle.

„ 20-21. The measurement of angles. Degrees

CHAPTER III.—TRIANGLES.

Lessons 22-24 Triangles and how to draw them.

„ 25-26. Some properties of triangles with practical methods of drawing certain angles, &c

CHAPTER IV.—FOUR-SIDED FIGURES.

Lessons 27-34. Construction and properties of (*a*) Squares, (*b*) Rectangles, (*c*) Parallelograms, (*d*) Rhombuses , with a short account of some of the properties of Parallel Lines.

CHAPTER V.—AREA.

Lessons 35-40 Meaning of word "area." Unit of area. Area of Squares, Rectangles, Parallelograms, Triangles, and Rectilinear Figures.

It is assumed that the pupil possesses some knowledge of the earlier Propositions of Euclid, Book 1.

ALLAHABAD, *August*, 1897. J. MURRAY.

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PRACTICAL GEOMETRY.

CHAPTER 1.—STRAIGHT LINES.

Lesson 1.

Straight Lines

It is not easy to draw a straight line on a slate or piece of paper, without using a straight edge or ruler to guide the hand; and if we are required to draw a *perfectly* straight line, we *must* use some instrument to assist us. It will, however, be convenient to be able to draw a line *approximately* straight, without using any instrument, guiding the hand by the eye alone. Although this is rather difficult at first, it becomes comparatively easy after some practice.

EXERCISE.

Draw, by eye alone, in your note-book, a number of *straight* lines each about three or four inches in length.

Lesson 2.

Straight Lines of given length

It is often convenient to be able to tell by eye alone, whether two lines are of equal length or not; and it is especially convenient to be able to draw a straight line which shall be equal, or very nearly equal

in length to another given straight line, estimating the correct length by eye alone.

EXERCISE.

Draw underneath the straight line in your note-book a number of straight lines, each equal in length to the given line estimating the correct length by eye alone.

LESSON 3.

Multiple Lines.

After acquiring facility in drawing by eye, a straight line, equal in length to a given straight line, it is comparatively easy to draw, by eye, a straight line, twice the length of a given straight line. The method is,—first draw a straight line, equal in length to the given straight line, then from one end of it, draw another equal straight line, so as to be in the same line with the first.

In the same way, a straight line, three, four or any number of times a given straight line may be drawn.

EXERCISE

Draw in your note book, underneath the given straight line, a number of straight lines, twice, three times and four times the length of the given line, estimating the correct length by eye alone

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LESSON 4.

Division of Straight Lines.

In Lesson 3, we saw how to draw straight lines any number of times the length of a given straight line. To be able to do this is important, but it is much more

important to be able to *divide* a given straight line, by eye alone, into two, three or more parts of equal length.

To divide a straight line in two equal parts is not difficult. The method is—put down the point of the pencil somewhere about the middle point of the straight line, then, carefully compare by eye, the lengths of the two parts on either side of the pencil point. If one part seems longer than the other, move the pencil slightly towards the longer part, and compare again, and so on till the two parts seem *exactly* equal. When this is so, make a mark, where the pencil point is. This mark divides the straight line into two equal parts.

When a line is divided into two equal parts it is said to be *bisected.*

To divide a straight line into three equal parts by eye, is slightly more difficult. The method is similar to that for division into two equal parts. Two pencils are necessary (any other pointed objects, such as pins, &c., will serve). Take one in each hand, and put their points, on two points in the line, so that the three parts into which the line is divided by these two points are all equal. Slight adjustments may be necessary, as in the previous case. When the parts seem exactly equal, marks should be made at the points. These marks divide the straight line into three equal parts.

When a line is divided into three equal parts, it is said to be *trisected.*

EXERCISES.

1 Bisect by eye, each of the lines in your note book.

2. Trisect by eye, each of the lines in your note book.

Lesson 5.

Division of Straight Lines.

A straight line can be divided into four equal parts, by first bisecting the whole line, and then bisecting each part. In a similar way a line can be divided into eight, sixteen, &c., equal parts.

A line can be divided in six equal parts by first trisecting the line, and then bisecting each part.

By again bisecting each of the six parts we divide the line into twelve equal parts. From twelve we can obtain, similarly, 24, 48, etc., equal parts.

By first trisecting the line and then trisecting each of the three parts, we get nine equal parts. By bisecting each of these parts, 18 equal parts, and by trisecting them, 27 equal parts are obtained. Thus we can by eye alone, easily divide any given straight line into any of the following equal parts, *viz.*:—2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 27, 36, &c.

We have seen that by *first* trisecting a given line, and *then* bisecting each of the three parts, we divide the line into six equal parts. Each part is thus one-sixth of the original line. Also, each part is one-half of one-third; so we see that one-half of one-third equals one-sixth ($\frac{1}{2}$ of $\frac{1}{3}=\frac{1}{6}$).

We could have divided the given line into six equal parts, by *first* bisecting it and *then* trisecting each part. In this case each of the six parts is one-third part of a half, hence we see, that $\frac{1}{3}$ of $\frac{1}{2}$ also$=\frac{1}{6}$. We conclude, then, that $\frac{1}{3}$ of $\frac{1}{2}=\frac{1}{2}$ of $\frac{1}{3}=\frac{1}{6}$.

This exemplifies, what is known from arithmetic, that in multiplication of vulgar fractions, the *order* in which the fractions are put down is of no consequence.

EXERCISES.

1. Trisect the line in your note book (marked A), and then bisect each part. Also bisect the equal line (marked B) and trisect the two equal parts. Compare the length of each part of A, with each part of B.

2 How many sixths of a line are contained in one-half, and one-third of the line respectively? What is the difference between $\frac{1}{2}$ and $\frac{1}{3}$?

3. Show how, by dividing a line into eight equal parts, we can see that $\frac{3}{4}=\frac{6}{8}$ and $\frac{1}{2}=\frac{2}{4}=\frac{4}{8}$; also by the same method, find the difference between $\frac{3}{8}$ and $\frac{3}{4}$; and between $\frac{1}{4}$ and $\frac{7}{8}$; and between $\frac{5}{8}$ and $\frac{1}{2}$.

4. Prove the following results geometrically —

(1). $\frac{10}{12}=\frac{5}{6}$.

(2). $\frac{5}{6}-\frac{1}{4}=\frac{7}{12}$.

(3). $\frac{2}{3}+\frac{1}{4}=\frac{11}{12}$.

(4). $\frac{5}{6}-\frac{3}{4}=\frac{1}{12}$.

Lesson 6.

Division of Straight Lines

It is comparatively easy to divide a given straight line, by eye, into any of the parts mentioned in the previous lessons. It is not so easy to divide it into *five* equal parts. It is, however, extremely important that we should be able to do so, with fair accuracy, for it will be frequently necessary to divide a line into *ten* equal parts, and to do that we must be first able to divide the line into *five* equal parts.

To divide a line into five equal parts by eye, we require four pins, which we must place on the straight line as accurately as we can by eye, so as to make the five parts into which the line is thus divided, as nearly equal as possible.

When the line is thus divided into five equal parts, ten equal parts are obtained, by bisecting each of the five parts.

EXERCISE.

Divide each of the straight lines in the exercises of Lesson 4, first into five equal parts by eye, then into tenths by bisecting each of the five parts.

Note.—This exercise is very important.

Lesson 7.

Comparison of Lines.

In the previous exercises, our main object was to determine, whether two straight lines were of equal length or not. If one of the lines is much longer than the other, we can tell easily enough that they are unequal, and also which is the longer. But when two lines are very nearly, but not *quite* equal, it is much more difficult to tell which is the longer, *especially if the lines are some distance apart.*

To shew that the lengths of lines can be more easily compared, when the lines are near each other, than when they are far apart, draw a straight line on the board about a foot long, and immediately underneath it, another straight line, about half an inch longer. It is easy to tell which of these two is the longer.

Now rub out the lower line, and draw another of the same length, about a foot below the first line. We can still tell, by eye alone, which is the longer line, although it is now more difficult, than it was when the two lines were near each other. Again rub out the lower line and draw another of the same length still farther away from the first line. We find it still more difficult to say which is the longer. If we go on drawing the lines farther and farther apart, we find that it becomes more and more difficult to detect the difference in their lengths.

It is especially difficult if the lines are drawn, not only far apart but in different directions.

Thus we see that *if* we wish to compare the lengths of lines easily and accurately, we must draw them as near to each other as possible.

This fact is well known. If we wish to compare the lengths of two pencils, for example, we place them alongside each other, adjusting one end of the one as close as possible to one end of the other. If the two pencils are not exactly equal in length, one will project a little beyond the other. In this way it is possible to detect a very small difference in their lengths. A difference equal to the thickness of a hair can be seen without much difficulty, and it is said that differences much smaller than this can be *felt*, even when they cannot be seen.

When one straight line is placed on another so that it lies along the other and has one of its ends coinciding with one end of the other, it is said to be "superposed" on the other. The lengths of two

straight lines, therefore, can be most *readily* compared, when the one is superposed on the other.

If two straight lines, equal in length, are superposed, we know that the ends of one will exactly coincide with the ends of the other, and conversely, **if when two straight lines are superposed, the ends of one exactly coincide with the ends of the other, the two straight lines are equal in length**

QUESTIONS.

1. Under what circumstances can the eye most readily detect slight differences in the lengths of lines.

2 What is probably the least difference in the lengths of two straight lines, which the unaided eye can detect ?

3. What is the test of equality in the lengths of two given straight lines ?

4. Is it possible that two straight lines should really have different lengths even when the unaided eye can detect no difference in their lengths ?

Lesson 8.

Comparison of Lines.

From the previous Lesson, we see that we can compare the lengths of any two straight lines with accuracy, if we can superpose the one on the other. *But it is often impossible to superpose one line on another*; one line on a blackboard for example, cannot be lifted up, and superposed on another line on the same blackboard, and two lines on the same sheet of paper are difficult to superpose. Indeed, the cases

where one line can be superposed on another are so rare, that we might almost consider this method of comparing lines useless in practice.

It would certainly be very nearly useless, if we had no means of surmounting this difficulty. But by means of a simple device, we find we can accurately compare the lengths of lines even though we cannot superpose them. **We take a third straight line, and superpose** it on each of the lines to be compared.

For example, by means of a piece of thread, we can compare the lengths of two straight lines on a black-board. By superposing the thread on one of the lines, we determine what length of thread has the same length as the line: by now superposing this length on the other line, we determine whether or not the *second* line is equal in length to the piece of thread. If the **same length** of thread **exactly covers both lines** we say the two lines are of **equal length**; if the same length of thread does not cover both lines, the lines are of unequal lengths.

EXERCISES

1 Use a piece of thread to test the equality in length of the lines drawn in your note book under Lesson 2

2. Similarly test the accuracy of the bisection of the lines in Ex 1, Lesson 4

LESSON 9.

Comparison of Lines

In comparing lines, we shall often find two, that are of *unequal* length, and may require to know *how much* longer or shorter the one is than the other.

To determine how many times one line is longer than another, we mark off on the longer line a succession of parts, each equal to the shorter line, in such a way, that the first part begins *at one end of the longer line*, the second part begins *at the end of the first part*, the third *at the end of the second* and so on, till the whole of the longer line is marked off, or till the part remaining is shorter than the shorter of the two lines.

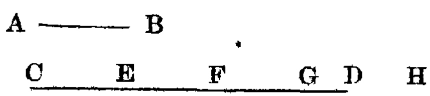


Fig. I.

Suppose we require to know how many times the line, CD, Fig. 1, is longer than the line, AB. Mark off on CD, parts CE, EF, FG, each equal in length to AB. Suppose GD is less than AB, and that GH (the dotted line, DH, is in the same straight line as CD) is equal to AB.

Then since CG is three times the length of AB, and CD is greater than CG, CD is greater than three times the length of AB. Also since CH is four times the length of AB, and CD is less than CH, CD is less than four times the length of AB. Hence CD is greater than three, and less than four times the length of AB. Thus we have found approximately how many times CD is longer than AB.

If G had exactly coincided with D, then CD would have been exactly three times the length of AB; if H had coincided with D, CD would have been four times AB. But since D lies *between* G and H, we

cannot find exactly how many times CD is longer than AB, but must *for the present* content ourselves with the approximate result, stated above.

When one line contains another line an *exact* number of times, its length is said to be an **integral multiple** of the length of the other line.

EXERCISE.

Determine how many times each of the lines marked B, C, D, &c., in your note book, is longer than the line A. State in the case of each line, whether it is an *integral* multiple, or not.

LESSON 10.

Comparison of Lines—Centimetre

The length of the line, A, in the exercise of Lesson 9, should be noted very carefully, for we shall have to use it very frequently, in our subsequent work. It has been given a special name, *which must be remembered.* It is called a '**CENTIMETRE**'

This is really a *French* word, but it has been adopted by English scientific people, and has been used so much in England, that we may now consider it an English word. We shall usually denote it by the letters "Cm"; thus, *one centimetre* will usually be written "1 Cm."; *two centimetres*, "2 Cms." &c.; a line four times the length of A is said to be "4 Cms. in length," or "4 Cms. long."

EXERCISES

1. Draw in your note book, a straight line 4 Cms. long
2 „ „ „ „ 5 „ „
3. Draw along the straight edge of a stout peice of paper, a straight line 10 Cms long marking off the Cms on it *very accurately*, and numbering them 1, 2, &c., up to 10.

4. Use this piece of paper to find the number of Cms. in each of the lines given in your note book, stating in each case if the number is *exact* or *approximate*.

Lesson 11.

Measurement of Lines—Scale.

We are said to **measure the length of a line** when we find (as in Ex. 4, Lesson 10) **the number of Cms. it contains;** thus we see that measuring the length of a line is the same thing as finding **how many times the line is longer than the 1 Cm line.**

A method of comparing the length of one line with the length of another was given in Lesson 9. But we see that when the line to be measured is long (line E., of Ex., Lesson 9, for example) the process of measurement becomes *long* and *laborious*. If the line to be measured were as long as the desk, for instance, the process of measurement, would, by that method, be excessively tedious. A line 100 Cms. long would require 100 applications of our 1 Cm. line. Each application would probably take at least 3 seconds. So to measure a line 100 Cms. long would take at least 300 seconds, or 5 minutes. If, however, we used the piece of paper with the 10 Cms. marked on it, (Ex. 3, Lesson 10) to measure the line 100 Cms. long, we should require *only ten* applications. And as we could probably apply the piece of paper 10 Cms. long, as rapidly as the line 1 Cm. long, the time required would now be *only one-tenth* of what it was before, or about half a minute.

We see therefore that in measuring *long lines* it is desirable to use a line much longer than 1 Cm. If the

lines to be measured are not more than 100 Cms. long, a line 10 Cms. long will be found a very convenient measuring line.

If the lines to be measured were several hundred Cms. long, it might be desirable to use a measuring line longer than 10 Cms., but we shall not in general have to measure lines so long as that.

The piece of paper with the Cms. marked on it may be called a **Scale**. A scale may be of any material. Paper is often used, though less frequently than wood.

When very accurate scales are required, steel or some other metal is employed.

In most scales the end of each centimetre is marked by a line drawn some distance across the substance of which the scale is made; and the piece of wood, or paper, with these lines drawn across it somewhat resembles a *ladder*; this is the reason why it is called a *scale*, for the word scale is closely connected with the Latin word *scala*, which means a *ladder*.

We see also that the 10 Cms. scale is as convenient for measuring lines *less than 13 Cms. long* as it is for lines longer than 10 Cms. For when applied to a line between 6 Cms. and 7 Cms. in length we see *at once* that the line is more than 6, and less than 7, Cms. long, since its extremity lies between the sixth and the seventh Cm. mark.

EXERCISE.

Use your 10 Cm. scale to find (to the nearest Cm.)

(1). The length of your pencil.

(2). „ „ your pen.

(3). „ „ your longest finger.

(4). The length of the thread which *just* goes once round your wrist.

(5). The breadth of the desk.

(6). " " form.

(7). The length of your space.

(A span may be defined as the distance from the extremity of the third finger to the extremity of the thumb, when the fingers are stretched *as far apart as possible.*)

LESSON 12.

Measurement of Lines—Millimetre.

The measurements which we made with our 10 Cm. scale, were not, as a rule, quite exact. In a few cases the length to be measured was an exact number of Cms., and in these cases we could tell the length exactly; but, in general, the length to be measured was *not* an exact number of Cms., and when that was so, we could measure the length only approximately.

The question than arises,--Is there no means of measuring lines more exactly? If, for instance, we find that a line is more than 7, and less than 8 Cms. long, must we be content with this? Can we not find out more exactly *how much* longer it is than 7 Cms. and how much shorter than 8 Cms.?

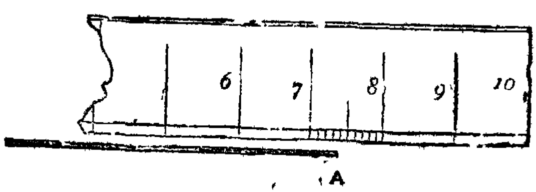


Fig. I.

Suppose, the end of the line to be measured, just reaches to the point A, in Fig. 1. We see that A is between the 7th and 8th centimetre mark, and nearer the 7th than the 8th. Is it not possible to estimate by eye, more or less correctly, how far the point A is from the 7th Centimetre? Yes, it is possible. If we divide the distance between the 7th and the 8th lines into 10 equal parts, we find that the point A is very near the fourth of these divisions, counting from the 7th, or the 6th, counting from the 8th line. Therefore the line is longer than 7 Cms. by *very* nearly 4 tenths of a Cm. So we might say its length is $7\frac{4}{10}$ Cms. or using decimals, we would write it 7·4 Cms.

We know that this is very nearly its true length. For we are quite sure that its true length is more nearly 7·4 Cms. than *either* 7·5 Cms, or 7·3 Cms. Hence our error in writing 7·4 Cms as its true length is *less* than $\frac{1}{10}$ of a Cm.

By this means we have succeeded in finding a value for the length, which is *much more accurate* than our first value (7 Cms.). In our first measurement the error was less than 1 Cm., but now we are sure, that the error is less than $\frac{1}{10}$ Cm. or ·1 Cm. **Thus by dividing the Cm into *ten* equal parts, we have reduced the error to *one-tenth* of what it was before.**

Each of these ten equal parts into which the Cm. was divided is called a **millimetre** (usually written "Mm." so that we have 1 Mm.$=\frac{1}{10}$ Cm. or ·1 Cm.

For convenience of measurement, Cm scales are usually made with *each* Cm. divided into ten equal

parts, *i.e.*, into millimetres* and we shall in future always use such scales, so that our measurement may be correct to *less than* 1 *Mm.*

EXERCISES

1. Re-measure by means of the Cm scale (divided to Mms) the lines in Ex. 4, lesson 10 (measurements to be made to Mms).

2. Re-measure (correct to Mms.) the lengths of the lines in the Exercise of Lesson 11.

3. Measure the length of the desk, by spanning.

4. Draw as accurately as possible, with scale and ruler, straight lines of the following length :—(1) 4·7 Cms , (2) 5 1 Cms., (3) 1 9 Cms , (4) 14 5 Cms , (5) 15·2 Cms., (6) 2·5 Cms., (7) 17·4 Cms.

Lesson 13.

Method of using the scale

We saw in Lesson 7, that the best way to compare the lengths of two straight lines, was to superpose the one on the other. Similarly, the best way to *measure* the length of a straight line, by a scale, is to place the graduated edge of the scale, *as near as possible* to the line we wish to measure.

You will see more clearly the advantage of doing this if you think for a moment, what we really do, when we measure the length of a straight line by a scale. We, really, find out *what length of the scale has exactly the same length as the line to be measured.* We determine *equality* or *inequality* in the length of two straight lines, the one, the line to be

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*Scales divided to Mms., may be obtained from the "Scientific Apparatus Manufacturing Company," Colonelganj, Allahabad.

measured, the other, some particular length along the edge of the scale; and we know that the best way to find out whether these two lines have equal length or not, is to *superpose the one on the other.*

Therefore in using a scale to measure the length of the line we must,

(1) **place the graduated edge of the scale as near as possible to the line to be measured.**

(2) **be very careful that one end of the scale exactly coincides with one end of the line to be measured.**

These two precautions must always be observed in measurements of length.

If we use a paper scale, we naturally find it easy enough to place the scale near the line to be measured. You might even say, that no one would think of doing otherwise, and to a certain extent this is true. But it is not so if we use a wooden scale. Wooden or steel scales possess several advantages; they can be made more accurate than paper scales, for they are not so liable to change their length through changes in the weather, and they are much more lasting. But they have to be used a *particular way* to give *correct* results. They are usually a millimetre or two in thickness; hence, if we place such a scale flat on the line, with its graduated face *uppermost*, the graduated edge of the scale will be at a distance of *a millimetre or two* from the line to be measured, being separated from it by the thickness of the scale. But we know that we cannot accurately compare the

lengths of two lines when such a distance lies between them. Consequently we see that we must place the scale on the line *with its narrow edge uppermost.* In this way, we can bring the graduated edge of the scale *quite close* to the line, whose length we wish to measure.

To measure a line, by means of a wooden scale, seems a simple enough operation, and so it is, but we see that there is a *right* and a *wrong* way of doing it. The object of these lessons is to train you in the right, the accurate way, and to enable you to avoid the wrong, the inaccurate way, of doing things. The scale is the first *instrument* we have had to use, and we have seen if we use it properly, it will enable us to find the lengths of lines very accurately, far more accurately than we could determine them by eye alone. But if we use it in the wrong way, we are almost as likely to estimate the length incorrectly, as we should be, had we no scale at all.

(Herein lies the difference between the *trained* experimenter and the *amateur* or novice. The trained experimenter has been taught to use all his senses, and instruments in the best, *i. e.*, the most accurate way. *By patient*, and *careful practice* he attains to such habits of accuracy, that it becomes almost impossible for him to make a mistake. The amateur on the other hand finds it the easiest thing in the world to make a mistake and that to avoid making mistakes he must spend a great deal of care and patience He should reflect, however, that *every trained experimenter was once a mere beginner*, and that every effort towards accuracy, helps him onwards. Habits of care and accuracy are gradually acquired and week by week he will find it becoming easier to avoid the wrong and follow the right.)

EXERCISES.

1. What two precautions must be observed in measuring the length of a line by means of a scale?

2. What special precautions have to be taken to ensure accuracy, when the scale used is a wooden one several millimetres thick?

3. What is the chief difference between an *amateur* and a *trained experimenter*?

Lesson 14.

Errors.

Errors in measurements may arise from two causes, defects in the *experimenter*, or defects in the *instrument*.

The experimenter may be *careless* or may use his instruments *in an improper manner*, and so obtain incorrect results. Errors arising in this way, however, are always avoidable. If it is too much to say that they can *always* be avoided, it is certain that every student who follows carefully the instructions given, and is resolved to do his best to avoid making mistakes, will in time practically succeed in doing so.

But errors may arise from defects in the instrument. We saw in Lesson 12, that if we used a scale divided only to *centimetres* we were liable to make an error in our measurements which might amount to as much as *half a centimetre.* We found, however, that by dividing the centimetre into ten equal parts, we could diminish the error to *one-tenth* of its previous value, our results being correct to about *half a millimetre.* In the figure of Lesson 12, the end of the line A came quite close to the 4th millimetre division (reckon-

ing from the 7th centimetre division towards the 8th). Consequently we gave 7·4 Cm. as the true length.

If, however, the end of A had *not reached quite up to the 4th millimetre division*, but had been somewhere between the 3rd and the 4th, we could no longer say that 7·4 Cm. was the correct length. All we could say is, that it must be *between* 7·3 *and* 7·4 Cms. We should be in error if we gave *either* 7·3 Cms. *or* 7·4 Cms. as the correct length, although our error would be small.

Small though it is, it is still an error, and the question arises,—How can we *avoid* or *diminish* it? We might try the same method as we used in Lesson 12, *i.e.*, we might divide the *millimetre* into *ten* equal parts, and see which of these minute sub-divisions lay nearest to the end of A.

Scales, however, are not made with such small divisions as these; millimetre divisions are the smallest in the scales generally used. But we can (as in Lesson 6 (*estimate by eye* to tenths of a millimetre, and then see *which of these sub-divisions lies nearest to the end of the line.* Although this may seem a somewhat unreliable method, it is yet found capable of yielding quite accurate results.

Suppose, then, we have estimated that the 7th subdivision (counting from the 3rd millimetre to the 4th), lies nearest the end of A. We know that the correct length is exceedingly nearly equal to 7·3 Cms. + $\frac{7}{10}$ of a Mm. But since 1 Mm. = ·1 Cm., ∴ $\frac{1}{10}$ Mm. = ·01 Cm., and $\frac{7}{10}$ Mm. = ·07 Cm. Thus we may write the correct length as 7·3 Cms. + ·07 Cms., *i.e.*, 7·37 Cms.

What is *now* the magnitude of our error?

If we are quite certain (and we can be, after some practice in estimating, by eye, to tenths of a millimetre) that the end A lies near the 7th sub-division, and not near the 6th or 8th, we may safely say that 7·36 Cms. is *too small* and 7·38 Cms. *too large* a value for the length; therefore our error cannot be more than ·01 Cm., *i. e.*, *one hundredth part of a Cm.* and may be less. Now ·01 Cm. is so small a length, that we may neglect it in comparison with the total length of the line, and say, that the *correct length of the line is 7·37 Cms.*

If we were to use *very* delicate scales, graduated to tenths of a millimtre, and were to examine the ends of the line with a magnifying glass, we might find that 7·37 Cms. was perhaps slightly inaccurate, and might be able, *just as above, to divide the tenth of a* millimetre into ten equal parts, and might find, for example, that 7·374 Cms. was a more correct value of the length. What would now be the magnitude of the error? It we were quite sure that the length was more nearly equal to 7·374 Cms. than to either 7·373 Cms. or 7·375 Cms., the error would be less than ·001 Cm., *i. e.*, the *one thousandth part of a centimetre.*

So, we see, that by dividing each division of our centimetre scale into millimetres we reduce the error to less than one millimetre; by estimating by eye to tenths of a millimetre, we further reduce the error to less than one-tenth of a millimetre; and that if we divide each millimetre division into tenths and use a magnifying glass, we might be able to estimate to

hundredths of a millimetre, *i. e.*, to thousandths of a centimetre. Thus by dividing the scale into finer and finer divisions, we reduce the error to the *one thousandth part of a centimetre.*

Now, although this error is very small, it is still an error, and our measurements are even yet slightly inexact, and the question arises, can we still further reduce the error? It is almost certain that we cannot, at any rate, by the method of sub-division. In the first place, it would be extremely difficult to divide the scale further. We have already supposed each centimetre divided into one hundred parts; if we now divided each of these small divisions into ten parts, we should require a good microscope to even see the lines, and it is extremely doubtful if we could estimate by eye correct to the tenth part of one of these excessively minute parts. So we may say, that the utmost limit of accuracy attainable by dividing a scale is about the one thousandth part of a centimetre, and to reach even this, we must use a magnifying glass and a very finely divided scale.

We shall find that for general work, a scale divided only to millimetres is the most useful, and although the error from using such a scale may be as much as the one hundredth part of a centimetre, we must rest content, reflecting that for our purposes, this error is so insignificant that we may quite neglect it.

EXERCISE.

Re-measure the lines in the exercise of Lesson 9, as also those in Ex. 4, Lesson 10, correct to the one hundredth part of a centimetre (estimate to *tenth of a millimetre*, by eye).

NOTE.—Professor Rowland, of Baltimore, has constructed an engine by means of which it is possible to divide a centimetre into *ten thousand* equal parts. The lines are so close together that they can only be distinguished by the aid of a very powerful microscope. Scales, however, are not divided so finely as this; other methods are employed, when very exact measurements of lengths are required. The simplest device is perhaps the *vernier*; by its aid, the error can readily be reduced to the one thousandth part of a centimetre. A more efficient apparatus is the Dividing Engine, by means of which the error can be reduced below the ten thousandth part of centimetre. A special form of dividing engine, used in engineers' workshops, is capable of measuring small length of two or three centimetres correct to about the *two hundred thousandth part of a centimetre.* This instrument is used for detecting *very slight* differences in the lengths of lines which are very nearly equal and is capable of detecting the extension in the length of a piece of iron (an inch or two in length) caused by merely bringing the hand *near* it.

CHAPTER II.—ANGLES.

Lesson 15.

How angles are made.

You probably know that two straight lines, which meet, and are not in the same straight line, are said to be *inclined* to one another; and in books on geometry it is said that "a plane angle* is the *inclination* to one another of two straight lines which meet but are not in the same straight line." Thus two straight lines which meet but are not in the same straight line are said to make an angle with each other.

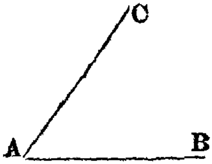


Fig. I.

For example, the two straight lines, AB, and AC, (Fig. I) meet, and are not in the same straight line. They, therefore, make an angle with each other.

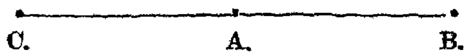


Fig. II.

The two straight lines, AB, and AC, (Fig. II) meet, but are in the same straight line. They, therefore, do *not* make an angle with each other.

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* The single word "angle" will in general be used in these Lessons, in place of the more precise phrases "plane angle," "plane rectilineal angle."

The two straight lines, AB, and AC (Fig. III), meet, but lie along each other, in the same straight line. They, therefore, do *not* make an angle with each other.

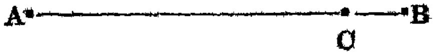


Fig. III.

Now suppose we draw (by means of a straight edge) two straight lines to form an angle.

Let us consider closely how this angle is made. We first put the straight edge on the paper (or board) in a suitable position, and draw one line. We then *turn the straight edge round* a little, taking care to keep it *on* the line first drawn, and having brought it to a suitable position, we draw the second line, meeting the first and making an angle with it.

So we see that to make the angle, we had to **turn** or **rotate** the straight edge, and we may consider the angle as having been produced by the **rotation of one of the straight lines round the angular point,** the other line meanwhile remaining fixed in position.

So we might consider the angle, BAC (Fig. I), as having been produced by the rotation or turning of the line, AC, about A, from the position it occupies in Fig. III, to the position it occupies in Fig. I.

It is important to think of angles as being produced or traced out in this way,—*viz.*, by the rotation about the angular point of *one* of the lines containing the angle, the other line remaining fixed,—for it enables us to realize how angles may become larger, and also that they are capable of growing to any size. The

further the movable line is *turned round*, the *larger* the angle becomes. Thus the size of an angle depends on the amount of rotation required to produce it. If a line revolving round a point moves from any one position to any other position, it *traces out an angle* and the amount of turning is a measure of the magnitude of the angle traced out.

Thus learn that angles are generated by the rotation of a straight line round a fixed point in the line.

EXERCISE.

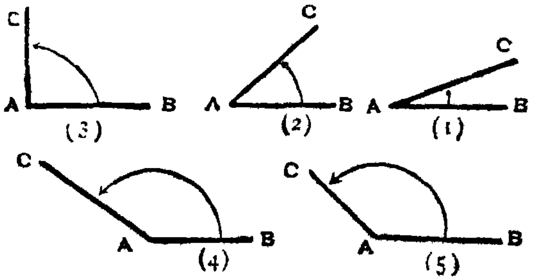
Determine by eye, the order of the given angles, as regards size.

Lesson 16.

Size of angles.

We saw in Lesson 15, that the size of an angle depends on the amount of the rotation required to trace out the angle, the greater the rotation, the greater the angle.

Let us consider the following angles, numbered (1) to (5), from this point of view.



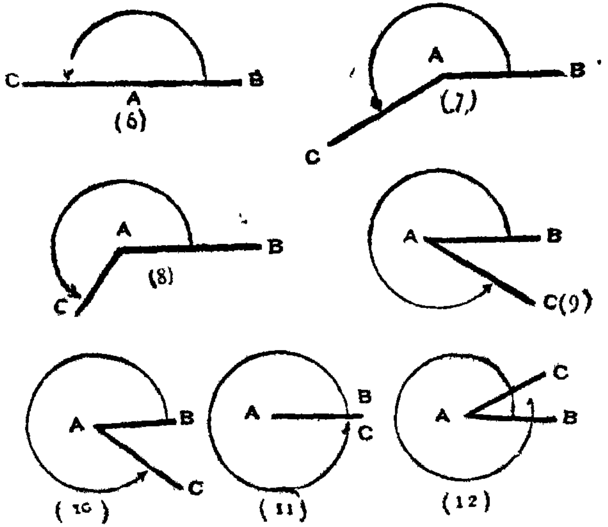
The angle (1) is small, as the rotation required to

move the line, AC, from AB to its final position is small. The angle (2) is larger, as AC has in this case been rotated farther. The angle (3) is larger still, as AC had to be turned still farther round to produce the angle. Similarly the angle (4) is greater than the angle (3), and the angle (5), than the angle 4.

Thus we see that the angle *increases with the rotation.*

But suppose the amount of rotation is increased still more, does the angle become still greater? Does the angle *always* increase with increase of rotation?

Let us consider the following angles, numbered (6) to (12), and see if the principle holds.



If the movable line, AC, is made to rotate from the

position it occupies in (5), it will after a slight rotation, occupy the position shewn in (6).

We have increased the rotation, have we also increased the angle? In (6) the lines AB, and AC, are in the same straight line. and therefore, by the definition of an angle given in books on Geometry, there is now no angle between them (see Fig. II, Lesson 15).

Now although we found in cases (1) to (5), that an increase of rotation gave an increase of angle, it seems that the increase of rotation from (5) to (6) not only did not increase the angle, but actually led to no angle at all. So the principle seems to hold no longer. The angle, it seems, does not always increase with the rotation.

Again, if AC is rotated still farther, it will occupy successively the positions shewn in (7) to (12). Does the angle between AB and AC continually increase as this rotation proceeds? On the contrary it seems to diminish; for, as we proceed from (7) to (12), the lines, AB and AC, seem to point more and more nearly in the same direction, and the angle between them seems to become smaller and smaller. If this were really so, we would have to conclude that increase of rotation produces an increase of angle only up to a certain point, but that after that point, a farther rotation gives a decreasing angle.

In these lessons, we shall not, as a rule, have to consider angles such as (6) to (12); we shall usually have to deal only with such angles as (1) to (5). Consequently, it is not of much practical importance to us at this stage, to consider whether the angles (7) to (12)

are greater than the angles (1) to (5) or not. But mathematicians, who have to consider all kinds of angles, say, that it is best to consider that an increase of rotation *always* produces an increase of angle, even in such angles as (7) to (12), and not only in angles like (1) to (5). They would therefore say that the angle (6) is greater than the angle (5), (7) greater than (6), (8) greater then (7), and so on.

In the case of (11) the movable line AC has gone exactly once round and lies along AB as it did at first. Even here, they would say an increase of rotation gives an increase of angle, and that consequently (11) is greater than (10), even although we might think that (11) was no angle at all.

In the same way (12) is greater than (11), and, of course, very much greater than (1), for to produce the angle (12) the movable line, AC, was turned more than once round, and therefore moved through a very much greater angle, than it moved through for the angle (1).

We, therefore, follow the mathematicians and say that the sizes of angles are *in every case* to be determined by the amounts of rotation which produced them and that the **greater the rotation, the greater the angle,** *no matter how great the rotation may be.*

One curious consequence of this is, that we cannot determine the size of an angle by merely looking at it. For example, the angle (12) looks very like the angle (1), and had we not known the amount of rotation, which produced the angle (12) (the rotation is indicated in the figure by the curved arrow), we might have said that the two angles were equal. But we know they are not equal, because their *rotations* were not the same. We seem, therefore, to require to know something about

the *previous history* of an angle, before we can say what its size is. The determination of the size of an angle is therefore not so simple an operation as it might seem to be.

EXERCISE.

Draw, by eye, angles equal to the given angles.

LESSON 17.

Equal angles.

When are two angles equal ? In considering this question let us confine our attention to angles *less than* (6) of Lesson 16 : how then can we test whether two such angles are equal or not ?

In Lesson 7, we found that two straight lines were of equal length, when they *coincided exactly on being superposed.* We use the same method in this case also, and test the equality of angles by *superposition.*

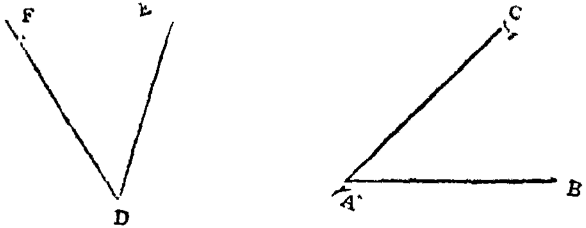


Fig. I.

Thus to determine whether the angle, FDE, is, or is not equal to the angle, CAB, Fig. I, we must (if possible) move the straight lines, DE, and DF, without altering the angle between them, to the lines, AC, and AB, so that D coincides with A, and DE lies along AB. If DF then lies along AC, the angles at D and A are equal. If DF lies between AC and AB, the

angle D is less than the angle A; if it lies outside AC, the angle D is greater than the angle A.

But a difficulty meets us here. We cannot, as a rule, move the lines forming an angle, so as to superpose them on another angle. Two angles on the same black board, for instance, cannot be surperposed; and it is seldom possible to superpose two angles drawn on the same sheet of paper.

We avoid the difficulty by the same device as we made use of in lesson 8. *We use a third angle* which we first make equal to one of the two angles to be compared. A piece of paper, for instance, which can be cut to any desired angle, is cut to *fit exactly* on one of the angles. This paper angle is then superposed on the other angle; if it fits exactly, we conclude that the two angles are equal to one another; if it does not fit the two angles are unequal.

EXERCISES.

1. Test by means of angles cut from paper, the results obtained, by eye, in the Exercise of Lesson 15.

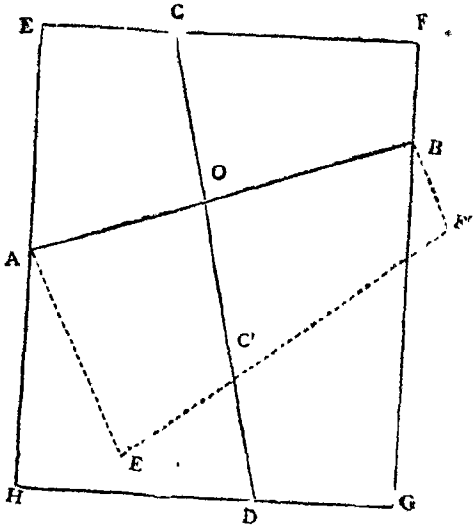
2 Test, as in Ex. 1, the angles drawn, by eye, in the Exercise of lesson 16.

Lesson 18.

Right angles.

Fold a thin sheet of paper, so that both parts lie quite flat on the table. The folded edge will be found perfectly straight if the folding has been done carefully. (Test by the straight-edge.)

Fold it again, by bringing the two ends of the folded edge together, then, flatten out on the table.



Thus suppose EFGH is the sheet of paper, and suppose it first folded along the line AB; the part above AB will, after being folded, lie over the part enclosed by the broken lines AE′F′B,—E will fall on E′, C will fall on C′, and F on F′. It is now to be folded a second time, by bringing B in contact with A, and then flattening out the paper on the table.

There are now four thicknesses of paper at the angular point, and four angles could be obtained by cutting the piece of paper along the folds.

If we, now, unfold the sheet of paper, and smooth it out flat on the table, we see that the creases, made by

the folds, from two straight lines, AOB and COC, crossing each other and forming four angles,—the four angles which we would have obtained by cutting the folded sheet of paper along the folds.

Regarding these angles we observe :—

(1) That they are all *equal*, for, when the sheet was folded twice, they were all superposed, and *fitted over each other exactly*.

(2) That the space around O, is exactly covered by the four angles.

(3) That, when *any two* of the angles are placed side by side, with their angular points in contact, and so that one edge of one lies along one edge of the other, then the *other two edges lie in the same straight line*: *e.g.*, if AOC and BOC are placed so that their edges touch along OC, their angular points being in contact, their other edges, OA, and OB, *lie in the same straight line*.

An angle of the same size as one of these equal angles has been given a special name. It is called a **Right Angle.** Thus we see, that **when a straight line** (as CO) **meets another straight line** (as AOB), **and makes the two adjacent angles** (as AOC and AOB) **equal to one another, each of the equal angles is a right angle.**

EXERCISES.

1. Fold a sheet of paper twice, as in this lesson ; form four paper angles by cutting along the folds. Verify their accuracy by placing them two by two, in contact [see (3) in this Lesson], and test (by the straight edge) whether their edges are in the same straight line or not.

2. Select one of the four right angles made in Ex. 1, and make four copies of it. Verify their accuracy, by finding

whether or not the four exactly cover the space round a point, [See (2) in this lesson].

3. Draw a number of right angles by eye alone.

Lesson 19.

Right angles.

The right angle is a very important angle; it will occur very frequently in our work; it is therefore very desirable that we should be quite familiar with it. We should *particularly aim at being able to tell, by eye,* whether any given angle is a right angle, greater than a right angle, or, less than a right angle.

By looking at the figure of Lesson 18, we see that the amount of rotation required to turn a straight line *once completely round* is **four times the amount of rotation required to turn it through one right angle.** This fact is otherwise expressed by saying that a line turns through *four* right angles in *one* complete revolution. If it made *two* complete revolutions it would turn through *eight* right angles, and if *three*, *twelve* right angles, &c.

Also if it has turned through two right angles, it has clearly completed half a revolution. When a straight line, therefore, is turned through half a revolution, it is in the same straight line as it was in, before it was turned, and is now lying on the opposite side of the point about which it turned.





Fig. I.

A **Fig. II.**

Thus if O A (Fig. I) is its first position, it will, after half a revolution, lie in the position indicated by the dotted line OA'. The line OA must rotate through two right angles to reach the position OA'. A farther rotation through two right angles (or, half a revolution) would of course bring it round to the position OA again.

A rotation through three right angles or three quarters of a revolution would bring the line into the position indicated by OA' (Fig. II).

EXERCISES.

1. What is the angle between the hour and minute hands of a clock when it shews three o'clock?

2. What fraction of a right angle is traced out by the hour hand of a clock in one hour? What angle is traced out by the minute hand in the same time?

3. What angle is traced out by the minute hand of a clock in ten minutes? What, in one minute?

4. In how many minutes will the minute hand of a clock trace out—

(1) a right angle?
(2) half a right angle?
(3) two-thirds of a right angle?
(4) one-fifth " " " " ?
(5) four-fifths " " " " ?
(6) two right angles?
(7) three " " ?
(8) four " " ?

5 What fraction of a right angle is traced out by the minute hand of a clock, while the hour hand moves over one minute?

6. What is the angle (fraction of a right angle) between the hour and minute hands of a clock when it shews—

(1) 12 minutes past 1 o'clock?
(2) " " " 2 o'clock?
(3) " " " 3 o'clock?

LESSON 20.

Measurement of angles—Degree.

In lesson 11, we saw that to measure a line we had only to find how *many times* it contained the standard line, one centimetre long.

In the same way we measure an angle by comparing it with some **standard angle.** We find *how many times* it contains that angle; the *number* so obtained, is called the **measure of the angle,** —**or,** sometimes, for shortness, **the angle.**

In the Exercises of Lesson 19, for example, we compared a large number of angles with the right angle, and found what fractions they were of that angle, *i.e.*, we used the *right angle* as our *standard angle*, and compared the other angles with it, just as, in measuring lengths, we used the centimetre as our standard length, and compared all other lengths with it.

There is, however, one serious *objection* to using the right angle as a standard. It is a *large* angle, larger than most of the angles we shall have to deal with. If we used it as our standard angle, we should find that the most of our angles would be denoted by *fractions*, *e.g.* $\frac{1}{2}$ of a right angle, $\frac{4}{5}$ of a right angle, &c., &c., and this would be extremely inconvenient.

To use a right angle as the standard angle would be as inconvenient as it would be to use a mile as the standard of length. Think what number would denote the length of your pencil, for instance, if the mile were our standard of length. It would be about $\frac{1}{6000}$, or ·000166, and our other lengths would be denoted by

equally inconvenient numbers. A mile is thus far too large to use as a standard of length and one of the *chief reasons* why the *centimetre* was selected as the standard of length, was that it was a *small* length, less than most of the lines to be measured, and the consequence is that the *numbers* expressing the length of the various lines we have to measure are, as a rule, *whole numbers* and *not fractions.*

In the same way, the right angle is too large as a standard angle. Some smaller angle must be selected.

The angle which has been selected is a very small angle indeed so small that *ninety* of them are required to make a right angle; this angle is called **"A Degree."**

Since there are ninety degrees in a right angle, there are of course 45 degrees in half a right angle, 30 degrees in one-third of a right angle, and so on.

Now we know that the angle between the hour and minute hands of a clock at one o'clock is one-third of a right angle. There are, therefore, 30 degrees between the two hands in that position.

Thus in five minutes the minute hand traces out 30 degrees, and in one minute it traces out 6 degrees.

N. B.—Degrees are generally denoted by a small circle at the top right hand corner of the number, thus "30 degrees" is written "30°."

EXERCISES.

1. Express the angles in Exercises 2, 3, 4, 5, and 6, of Lesson 19, in degrees.

2. How many degrees does a weathercock turn through in moving from due East to due West?

3. How many degrees does the earth turn through—

(1) in 24 hours?
(2) „ 1 hour?
(3) „ 4 minutes?
(4) „ 6 hours?
(5) „ during the time the school is open?

4. If one mile were selected as the unit of length, what number would denote—

(1) 4 furlongs?
(2) 880 yards?
(3) 44 yards?
(4) 1 yard?
(5) 1 inch?
(6) 10,560 feet?

N B—Use *decimals*, for fractional numbers.

5 If 1 Cm is taken as the unit of length, what number denotes—

(1) 1 inch?
(2) 1 foot?
(3) 1 yard?
(4) 1 mile?

Note—2 54 Cm. = 1 inch (*very* nearly).

6. Given that one lakh of Cms. = one Kilometre,—

(1) How many kilometres are there in one mile?
(2) „ „ miles „ „ „ „ kilometre?

Lesson 21.

Measurement of angles—Protractor.

In Lesson 11 we found we could measure the length of lines, and also draw lines of given lengths, much more easily if we used a scale than if we used only a single centimetre.

In the same way, to enable us more readily to draw any required angle, or measure any given angle, we use an instrument called a **Protractor.*** This is merely a semi-circular piece of metal, or wood, with a succession of marks round its curved edge, at equal small distances apart, such that the angle between the straight lines joining any two neighbouring marks to the centre of the semi-circle is *one degree.*

To draw an angle containing any given number of degrees, place the protractor on the paper, and mark on the paper—

(1) the point where the centre of the protractor rests.

(2) „ „ „ „ end of the line marked "O" rests.

(3) „ „ „ „ end of that line rests, whose number is the same as the number of degrees required; then lift the protractor from the paper, and use the straight edge to draw a straight line from each of the last two marks to the centre.

2. **To find the number of degrees in any given angle**, place the protractor on the angle, so that its centre is on the angular point, and its zero line coincides with one side of the angle. Then note the number of the mark on the edge of the protractor, *nearest* to the other side of the angle. This number is the number of degrees in the angle.

---

* Protractors may be had from the "Scientific Apparatus Manufacturing Co.," Colonelganj, Allahabad.

If the straight lines containing the angle do not extend beyond the edge of the protractor, but are completely hid by the protractor when placed on the angle, the containing lines should be produced a little so as to shew at the edge of the protractor. In producing the containing lines, *great care* must be taken to make the part produced to lie *in the same straight line* with the original line.

It will frequently happen that the angle to be measured does not contain an *exact* number of degrees. When this is so, an attempt should be made to estimate *by eye*, to the *tenth part* of a degree, just as, in Lesson 14 we estimated lengths of lines by eye to the *tenth part* of a millimetre,

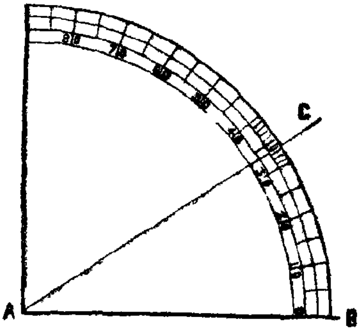


Fig. I.

For example, the angle BAC (Fig. 1) is greater than 33° and less than 34°. By eye, it may be *estimated* to be 33·7°, since it seems to lie beyond the 33rd degree by

seven-tenths of the distance between the 33rd degree and the 34th.

NOTE.—Protractors are sometimes made rectangular in shape.

EXERCISES.

1. Use protractor and straight edge, to draw the following angles :—90°, 60°, 50°, 45°, 37°, 30°, 18°, 10°, 6°, 3°, 2°.

2. Similarly draw the following angles,—60°, 100°, 120°, 135°, 158° 170°, 176°, 178°.

3. Use the protractor to determine the number of degrees in each of the angles of the exercise of Lesson 15.

CHAPTER III.—TRIANGLES.

LESSON 22.

Triangles, and how to draw them

SUCH figures as A, B, C, D, (Fig. I.) are called triangles.

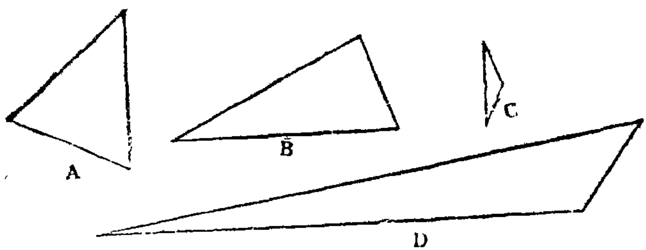


Fig. I.

The word **triangle** literally means "three angles," and, we see, there are *three angles* in each of the above figures. It is usual, however, in defining a triangle to refer only to its *sides*, making no mention of its angles. Thus,

A triangle is defined as a **closed rectilineal figure which has three sides.**

It is a **closed figure,** for one side begins where another ends, and it is impossible to pass, *on the paper* from any point within the triangle, to any point outside it, *without crossing a side* of the triangle.

It is called a **Rectilineal* figure** for it is made up of straight lines.

* Rectilineal means, "made of straight, or, right lines."

It has **three sides**, and therefore, *three angles*, and is thus, called a *triangle*.

Now, to draw a triangle, we must know its size and shape and the question arises, what does its size and shape depend on?

Clearly on the **lengths of its sides**, and the **sizes, of its angles.**

Must we, therefore, know the length of *each* side, and the size of *each* angle to be able to draw the triangle ?

Consider a simple case. Suppose we know that two adjacent sides of a triangle are 4, and 5 Cms., respectively, and that they are inclined to each other at an angle of 40°. Is this knowledge *sufficient* to enable us to draw the triangle ?

Clearly, it is, for we can easily draw (by means of our scale and protractor), two lines of 4, and 5, Cms., respectively, inclined to each other at an angle of 40°.

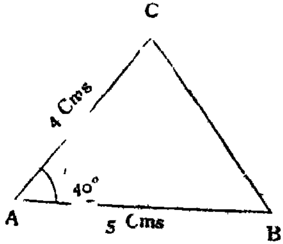


Fig. II.

Let AB, and AC (Fig. II) be these lines. Now since AB and AC are two of the sides of the triangle required, the third side must be the straight line BC. If we now draw the straight line BC, the triangle is complete.

From this we learn that to draw a triangle we need

to know only two of its sides and the angle, between them.

Hence we can draw triangle, even, although we do not know the length of *every* side, and the size of *every* angle.

EXERCISES

1. Draw the triangles of which the two sides, and included angle are given in the following table . —

No of triangle	1	2	3	4	5	6	7	8	9	10
Length of sides (in Cms)	1&5	3&5	6&8	8&8	3&8	3& 4 24	6& 2 5	12& 13	5 7& 4 3	2 7& 6 1
Included angle (in degrees)	37	53	68	44	60	45	90	22 6	12	57

2. Use scale and protractor to measure the length of the third side, and the size of the other two angles, in each of the above triangles

3 Verify that the sum of the lengths of *any* two sides of the above triangles, is greater than the length of the remaining side.

4 In each of the above triangles find the sum of the three angles.

Lesson 23.

Triangles, and how to draw them.

In the last Lesson, we saw that we could construct a triangle, if we knew the lengths of two of its sides, and the size of the included angle.

It is equally clear that we can construct the triangle, if we know the length of only one side provided we know also the sizes of the two angles which the other sides make with this side.

Suppose the given side is 5 Cms., and the two angles are 30° and 60°, respectively ; we first draw a straight line 5 Cms. long, as AB, (Fig. I).

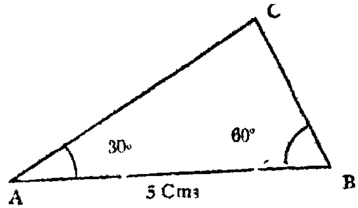


Fig. I.

We then draw through A and B, two lines making angles of 30° and 60°, respectively, with AB, as in the figure. These lines lie along the other twe sides of of the triangle, and the point, C, where they meet is, therefore the third angle of the triangle. Thus, ABC is the required triangle, for it has one side, AB, 5 Cms. long and two adajcent sides making angles of 30° and 60° with AB.

Hence we can construct *any* triangle if we know the length of one of its sides, and the sizes of the two adjacent angles.

EXERCISES

1. Construct the triangles of which one side, and the two adjacent angles are given in the following table —

No of triangle	1	2	3	4	5	6	7	8	9	10
Length of one side (in Cms)	3	4·3	8	8	8	2	6 7	2	3	5
Size of Adjacent angles.	40° & 50°	75° & 15°	30° & 30°	60° & 60°	40° & 70°	120° & 25°	100° & 45°	170° & 0°	90° & 60°	90° & 30°

2. **Verify that the greater of any two of the angles of the above triangles has the greater side opposite to it; also, that when two angles of a triangle are equal, the sides opposite to the equal angles are also equal.**

3. Verify from measurements of the 4th triangle of Ex. I,

(1) That when each of two angles of a triangle is 60°, the remaining angle is also 60°, (the triangle is *equiangular*),

(2) that when a triangle is equiangular, it is also equilateral, *i e* has all its sides of equal length.

4. Verify from measurements of triangles (9) and (10) of Ex. I, that if two sides of a right angled triangle make an angle of 60 with each other, the length of one side is double the length of the other.

Lesson 21.

Triangles, and how to make them

We have seen that a triangle can be constructed, (1) if one side and two adjacent angles are given, (2) if two sides and the included angle are given.

We shall see, in this Lesson, that a triangle can be constructed, if the **lengths of its three sides are given**.

In this case, who do not know any of the angles of the triangle; consequently, the problem is slightly more difficult, than the problems considered in the previous two Lessons.

Thus, suppose, we are required to draw a triangle with sides of 5, 7, and 8 Cms., respectively; we can easily draw *one* side, but how shall we draw *the other two*? In what *direction* shall be draw the lines, that they meet at the proper distances, from the ends of the first line?

Suppose we draw a straight line, AB, (Fig. I), 8 Cms. in length.

This is one side of the triangle, but *in what direction* does the third angle of the triangle lie? We do not know; we only know that it must be 5 Cms. distant from A, and 7 Cms. from B.

In order to find where the third angle, C, must lie, we use a device, which is very common in mathematics and which is given a special name. It is called, the method of **Intersection of Loci**.

We shall understand the method, by seeing its application to our problem.

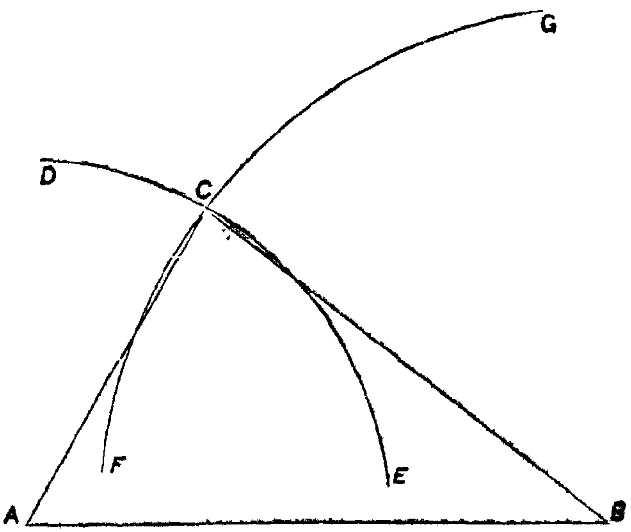


Fig. I.

Since C lies at a distance of 5 Cms. from A, let us first think of *all the points on the paper that are 5 Cms. distant from* A. We know that all such points lie on a **circle about A as centre** whose radius is 5 Cms. *All* points *within* that circle are *less* than 5 Cms. from A, and *all* points *without* the circle are *more* than 5 Cms. from A.

This circle is called by mathematicians the **Locus** of points 5 Cms. distant from A. (Locus literally means *place* or *position*,- hence our word, *locality*, --and in our case *Locus* means the *place* or *locality*, where all points, distant 5 Cms. from A, are to be found).

Now we know that C must be *somewhere* on this circle, although we do not yet know the *precise spot*.

Let us draw* this circle DCE (Fig. I),- in the Fig. only a part of the circle is shewn.

In the same way, we know that C lies **on a circle round B as centre**, whose radius is 7 Cms for this circle is the Locus of *all* points on the paper whose distance from B is 7 Cms. Let us draw this circle, FCG (Fig. I.).

Now since C lies on *both* these *Loci* (*Loci* is plural of *Locus*), it must be at a point common to both, *i.e.* at a point where one circle cuts the other. Thus, C is a **point of intersection of the two Loci.**

Having found C, we have now only to join C to A and B, and the triangle is complete.

---

* To draw the circle a pair of compasses will be required.

EXERCISES.

1. Use scale and compasses to draw the triangles whose sides are :—

(1) 4, 5, and 6	Cms.	respectively.
(2) 3, 5, and 5	,,	,,
(3) 6, 8, and 10	,,	,,
(4) 6, 8, and 8	,,	,,
(5) 8, 8, and 8	,,	,,
(6) 7·3, 5·9, and 4	,,	,,
(7) 4 5, 3·6, and 1·1	,,	,,

2. The circles round A and B, in this Lesson, intersect in *two* points. *Which* of these is to be taken as the third angle of the triangle ? If *both* are taken, and *two* triangles formed, in what respects do the two triangles differ from each other ?

3 Verify, by measuring the angles of triangles (2) and (3) in Ex. I, (above) that doubling *each* side of a triangle, leaves the angles unchanged.

LESSON 25.

Properties of triangles.

Although triangles may have a great variety of shapes, some small, some large, some with long sharp angles, some with short blunt angles, (see the triangles, of Fig. I, Lesson 22), there are yet a considerable number of points, in which all triangles are alike. These *points of resemblance* are called **Properties of the triangle.**

We shall now consider a few of these properties.

1. Referring to Exercise 3 of Lesson 22, we see that *any* two sides of the triangles of that exercise, are together greater than the third. If we were to draw triangles of all manner of different shapes, and sizes, and were to get on doing so for years, without

drawing *two* triangles alike, we should find in every one of these triangles, that **any two sides** were **together greater than the third.**

2. We saw in Exercise 4 of the same Lesson, that the sum of the three angles of the triangles was 180°, very nearly. This is another point, in which all triangles resemble each other. The sum of the three angles of *every* triangle is 180°. [It is really 180° exactly; our results were slightly inaccurate, because of slight errors in our measurements; the more accurate we make our measurements, the more nearly will the sum be found to approach 180°.]

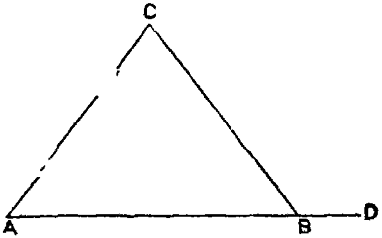


Fig. I.

3. From this result, we can easily shew that if one of the sides of a triangle is produced, *e. g.*, the side AB of the triangle ABC, (Fig. I.), **the exterior angle CBD is equal to the sum of the interior and opposite angles** at A and C.

For the angles at A and C together with the angle ABC make up 180°, and the angle CBD, with the angle CBA, makes up 180°. Hence the angle, CBD, is equal to the sum of the angles at A and C.

4. Another property of triangle is shewn in Ex. 2, Lesson 23, where we saw that **the greater of two**

unequal angles of a triangle has the greater side opposite to it.

These are a few of the properties of triangles which it will be useful to keep in mind.

It will be well also to remember that **each angle of an equilateral triangle is 60°.** (See Ex. 3, Lesson 23.)

EXERCISES.

1. Produce all the sides of a triangle and verify that each exterior angle of the triangle is equal to the sum of the two interior opposite angles.

2. From the same triangle verify that the sum of the exterior angles of a triangle is equal to four right angles.

3. What relatron must hold regarding the lengths of three straight lines that it may be possible to form them into a triangle ?

How many * triangles can be formed from six rods whose lengths are 1, 2, 3, 4, 5, and 6 Cms. respectively ? What are the lengths of the sides of the various triangles that can be formed ?

LESSON 26.

Practical Method of

(1) BISECTING ANY ANGLE,

(2) DRAWING CERTAIN ANGLES WITHOUT USING A PROTRACTOR.

(3) DRAWING THROUGH A GIVEN POINT A LINE PERPENDICULAR TO A GIVEN LINE.

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*It is not hard to shew that if there are n rods whose lengths are 1, 2, 3, &c., up to n Cms. respectively, the number of triangles that can be formed is

$\frac{m(m-1)(4m-5)}{1.2.3}$ when $n=2m$ and $\frac{m(m-1)(4m+1)}{1.2.3}$ when $n=2m+1$; m being any integer.

(1) Draw any isosceles* triangle, as ABC (Fig. 1) in which

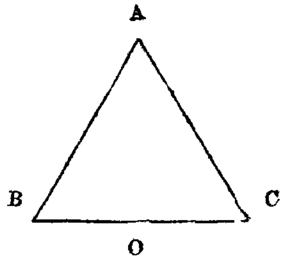


Fig. I.

the sides, AB and AC, are of equal length; if we cut out the triangle very neatly, and fold it so that the angle C falls on B, we know that the side BC will be divided into two equal parts by the fold and that the fold will be at right angles to BC; (see Lesson 18). Also if the cutting out and folding be accurately done, the fold will pass through A, and the appearance of the folded triangle will be as in Fig. II.

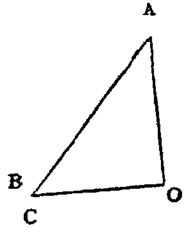


Fig. II.

The triangle will be divided by the fold into two parts which cover each other exactly; thus the fold

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*A triangle is said to be *isosceles* when two of its sides have the *same* length.

bisects the angle A; hence we learn, that the straight line through the intersection of the two equal sides of an isosceles triangle, perpendicular to the base, bisects both the base and the angle which it passes through. We also learn that the straight line, which bisects the base of an isosceles triangle perpendicularly, passes through, and bisects the vertical angle.

Thus to bisect any given angle, as AOB (Fig. III.), use the compasses to measure off any, equal, lengths, OC, and OD, along the containing lines, OA, and OB. Join the ends of these lengths by a straight line, and find (by trial with the compasses) the middle point, E, of the joining line. The line joining the point E, to the angular point O, will bisect the angle AOB.

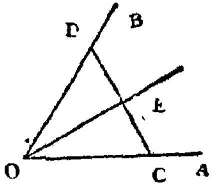


Fig. III.

In this way, any angle can be bisected; the only instruments required being, a straight edge, and a pair of compasses.

(2) Hence by drawing a right angle, and dividing it into two, and four, equal parts and combining the parts in various ways, we obtain the following angles, *viz.*—

90°, 45°, 22·5°, 135°, 112·5°, and 67·5°

using only our straight edge, and compasses.

Similarly, by straight edge and compasses, we can draw the following angles, *viz.*—

60°, 30°, 15°, 7·5°, 75° and 37·5°.

Again by combining these angles with the former, we obtain

150°, 105°, 82·5°, 172·5°, 127·5°, 120°, 52·5°, 165°, 142·5° and 97·5°.

(3) We may find it occasionally useful to be able to draw a straight line through a given point, in a direction perpendicular to a given straight line, without using a protractor.

The method is given in Euclid, Book I., Props. 11 and 12, to which reference should be made.

EXERCISES.

1. Verify, by drawing, cutting out, and folding, an isosceles triangle that the perpendicular bisector of the base (1) passes through the vertical angle, (2) bisects it

2. Verify by drawing two intersecting straight lines and bisecting *each* of the angles between them, that the two bisecting lines are at right angles to each other.

3. Bisect each of the given angles

4. Draw the following angles, by ruler and compasses alone, *viz* —60°, 30°, 15°, 75°, 90°, 105°, 120°, 150°.

5. Draw straight lines through the given points, perdendicular to the given straight line.

CHAPTER IV.—FOUR-SIDED FIGURES.

LESSON 27.

Square.

In Chapter III, we considered some of the properties of three-sided figures, or triangles; in this chapter, we shall examine some four-sided figures, and see what we can learn about them.

The first four-sided figure we shall consider, is one which is drawn in the following way:—

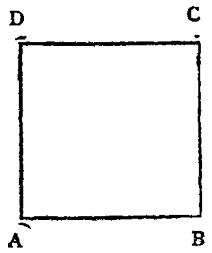


Fig I.

A straight line, AB (Fig. I), is first drawn. From the ends of this line two straight lines, AD, and BC, *each equal in length to* AB, are drawn towards the same side of AB, *and at right angles to it.* Finally a straight line is drawn from D to C and the figure is complete. A four-sided figure, drawn as above, is called a **square.**

We know from the method of constructing the figure that at least *two* of its angles, *are right angles,* and that at least *three* of its *sides have the same length.*

By carefully going through the following Exercises, we shall learn more about the properties of squares.

EXERCISES.

1. Which three of the four sides of the square, Fig. I, of this Lesson, do we *know* to have the same length? Which two of its four angles, do we *know* to be right augles?

2. Construct as in this Lesson, squares with sides of the following lengths *viz.*, 2 Cms., 4 Cms., 3 Cms., 6 Cms, 2 7 Cms., 5·4 Cms., 1 Cm

3. Measure the lengths of the last drawn sides in each of the squares in Ex. 2.

4. Measure the angles which the last drawn side makes with the other sides, in each of the squares in Ex. 2.

5. Measure the lengths of the two diagonals (the straight lines joining opposite corners) in each of the squares in Ex. 2.

6. Measure the angles between the two diagonals in each of the squares of Ex 2

7 Measure each of the two angles into which each angle of the square is divided by a diagonal.

8 Measure the lengths of the parts into which each diagonal is divided by the other.

Lesson 28.
Properties of squares.

NOTE—In this Lesson the teacher should endeavour to get the class *to realize for themselves*, the various properties of squares, which the Exercises of Lesson 27 are intended to illustrate. He should lead them to *find for themselves* the answers to the following questions —

What did we learn regarding the length of the fourth side in each of the squares in Ex. 2, Lesson 27?

What did we learn regarding the sizes of the angles in each of these squares? What, regarding the lengths of the diagonals; and the angle between them? What regarding the lengths of the parts into which each diagonal is divided by the other; what regarding the

sizes of the two angles into which each angle of the square is divided by the diagonals ?

EXERCISES

1. Which of the given four-sided figures are squares ?

2. The diagonals of certain squares are 3, 5, 4, 2·8, 4·3, and 7 Cms., respectively ; draw the squares.

3. Measure the lengths of the sides of the squares in Ex. 2 (of this Lesson)

4. What are the quotients,—length of diagonal, by length of a side, in the case of the squares in Ex. 2 of Lessons 27 and 28 ?

5. Write down *all* the properties of squares, with which you are acquainted.

Lesson 29.

Rectangle.

If the two equal straight lines, AD, and BC, (Fig. I, of Lesson 27), were made either longer, or shorter than AB, still remaining equal to each other, and perpendicular to AB, the figure would be what is called a **Rectangle.** (It is sometimes called an *Oblong*).

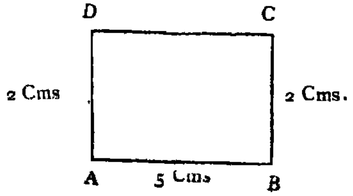


Fig. I.

Suppose we are required to construct a rectangle, of which one side is 3 Cms., and the two perpendicular sides each 2 Cms.

We, first, as in Lesson 27, draw a straight line, AB, (Fig. I.) 3 Cms. in length ; then at its ends, draw

two perpendicular straight lines AD, and BC, each 2 Cms. in length. These lines form three sides of the figure; the fourth is the straight line joining D to C.

We shall see in what respects this figure differs from a square and what properties it has in common with a square, as we proceed.

EXERCISES.

1. Construct, as in this Lesson, rectangles whose sides are respectively, 5 and 4, 2 and 4, 3 and 4 5, 4 and 1, 6 and 5, 3 and 7 Cms in length.

2 Measure the last drawn side in each of the above rectangles

3. Measure the angles which the last drawn side makes with the other sides in each of the above rectangles

4 Measure the lengths of the two diagonals in each of the above rectangles.

5. Measure the angles, between the two diagonals, in each of the above rectangles

6. Measure each of the two angles, into which each angle of the above rectangles is divided by a diagonal.

7. Measure the lengths of the parts into which each diagonal is divided by the other, in each of the above rectangles.

LESSON 30.

Properties of Rectangles

From the exercises of Lesson 29, the class should be led to find out for themselves, the various points in which rectangles differ from squares, also the points in which they agree.

When they have thoroughly mastered the results of their measurements, they should be asked to write in parallel columns in their note-books a statement of the corresponding properties of the two figures.

EXERCISES.

1. Which of the given four-sided figures are rectangles?

2. Measure the angles which each diagonal makes with each pair of opposite sides in each of the rectangles of Ex. 1, Lesson 29.

3. Draw the rectangles whose diagonals have the lengths and are inclined to each other at the angles, given below :—

No. of rectangle	1	2	3	4
Diagonal (in Cms)	4	6	5	4 6
Angle between the diagonals	60°	45°	57°	70°

4. Draw the rectangles, whose diagonals make given angles, with sides of given lengths, as in the following table :—

No of rectangle	1	2	3	4
Angle between a diagonal and a side	30°	40°	22 5°	85°
Length of the side	5	6	5 54	2

5. Compare the sizes of the angles, which the diagonals of the rectangles of Exs 3 and 4 make with the sides and with each other.

Lesson 31

Parallel Lines.

Looking at the results of our measurements in Exs. 2, and 5, Lesson 30, we see that the angles which a

diagonal of a rectangle makes with a pair of opposite sides are equal.

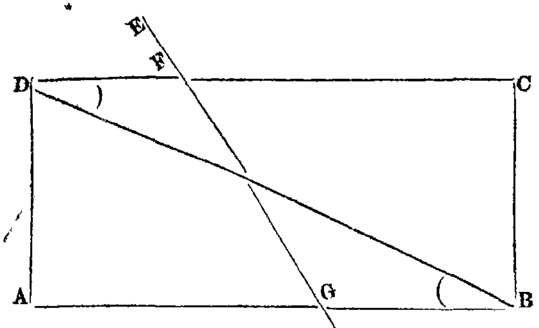


Fig. I.

For example the diagonal DB of the rectangle, ABCD, (Fig. I) makes the angles ABD and BDC, with the pair of opposite sides, AB and DC. Our measurements shew that these are *equal* angles.

Now let us draw *any* other straight line, such as EFGH, *to meet these opposite sides*; we find on measuring the angles CFG and FGA, that they also are *equal to one another*.

If we draw the line in *any* other position, or direction, we shall still find, that it makes equal angles with the pair of opposite sides.

Such angles as CFG and FGA, lying on *opposite sides of the line* EFGH, are usually called **Alternate angles.**

Now when a pair of straight lines are such that a straight line meeting both, makes **the alternate angles equal** to one another, we say that the pair of lines are **parallel to each other.**

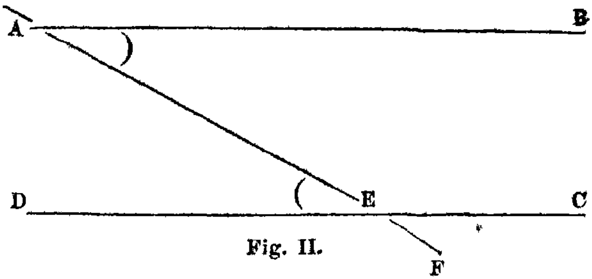


Fig. II.

For instance, the straight lines AB and CD, (Fig. II) are such that the straight line AEF, meeting the two lines, makes the *alternate angles* BAE, and AED, *equal to one another.*

The two straight lines AB and CD are therefore said to be *parallel to each other.*

Since the angles AED and CEF are equal to one another, the angles CEF and EAB are also equal to one another; hence *if the lines* AB *and* CD *are parallel, the straight line* AEF *meeting these lines makes the* **exterior angle FEC equal to the interior opposite angle on the same side, EAB.**

1. The above enables us to test whether two given lines are parallel or not.

To do so we draw *any* straight line to meet both and then measure the alternate angles. If *the alternate angles are equal*, the lines are *parallel*, if they are not equal, the lines are not parallel.

2. It also indicates a method of *drawing a straight line through a given point parallel to a given line.*

To do so, we first draw any straight line to meet the given line, and pass through the given point. Then

we use our protractor to draw a straight line through the given point, so as to make the *alternate angles equal to one another.*

To draw parallel lines, however, it is not necessary to use a protractor; it is sufficient to use a piece of metal, wood, or paper cut to form *any* angle. These pieces of metal, &c., are usually made triangular in shape, with one of the angles a right angle, as in Fig. III.

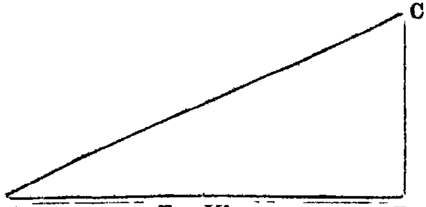


Fig III.

The method of using the instrument* to draw a straight line through a given point, parallel to a given straight line, will appear from the following example:—

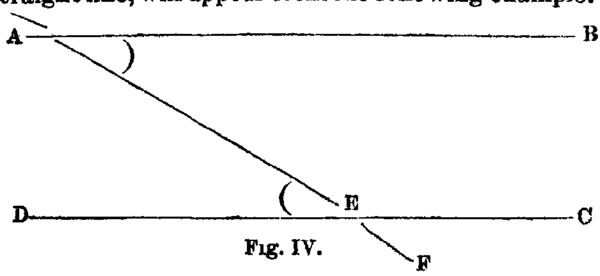


Fig. IV.

Suppose we are required to draw through the point E (Fig. IV) a straight line parallel to the straight line AB (Fig. IV) by means of a template (shewn in Fig. III).

* This instrument is sometimes called a *Template.*

(1) Adjust one edge of the instrument (say, AC, Fig. III) to coincide with the given line, AB, Fig. IV, so that the other edge (AB, Fig. III) passes through the given point, E (Fig. IV).

(2) Draw a line along the edge AB of the instrument; this line (AEF, Fig. IV) will pass through E and should be continued some little distance beyond E (as, EF).

(3) Move the instrument over the paper till the corner A of the instrument is at the point E, (Fig. IV) and adjust till the side AB of the instrument, coincides with the line EF.

(4) Draw a line along the side AC of the instrument.

This line, (EC, Fig. IV), will pass through E and will be parallel to AB (Fig. IV). It may be prolonged as far as is necessary in either direction by means of the straight edge.

Thus we have drawn the line DEC (Fig. IV) through the given point E parallel to the given line AB.

The template may also be used *to test the parallelism* of two given straight lines.

Suppose we are given the two lines, AB and DC, (Fig. IV), and wish to test whether they are parallel or not. We first adjust one edge of the instrument to coincide with one of the given lines (say AB); we then draw a line along the other edge of the template to meet both the given lines (say the line AEF, Fig. IV); lastly we superpose the angle A of the template on the alternate angle (AED, Fig. IV) and test whether the alternate angles are equal or not. If they are equal, the lines are parallel, if not, they are not parallel.

EXERCISES.

1. Use template (or protractor) to determine which of the given pairs of lines are parallel.

2. Verify that opposite sides (1) of the squares in Ex. 2, Lesson 27, (2) of the rectangles in Ex 1, Lesson 29, are parallel.

3 Use template (or protractor) to draw straight lines through the given points parallel to the given straight lines.

4. Draw through each of the given points two straight lines, which shall be respectively parallel to the two given intersecting straight lines.

5. Draw two parallel straight lines of equal length, and verify that the straight lines joining their ends towards the same parts are also equal and parallel.

6 Through the given point O draw straight lines to each of the points A, B, C, D, &c, in the given straight line; measure the angles at A, B, C, D, &c., and arrange them in decreasing order of magnitude.

7. Verify that if two sides of a triangle are *very long*, compared with the base or third side,

(1) The angle between the two longer sides is very small.
(2) The exterior angle at the base is very nearly equal to the interior opposite angle at the base.
(3) The sides are *very nearly parallel*

LESSON 32.

Parallelograms.

In Ex. 4, Lesson 31, two lines were drawn through given points, so as to be respectively parallel to given pairs of intersecting lines. A number of four-sided figures were thus produced, having their *opposite sides parallel.*

Each of these four-sided figures is called a **parallelogram.**

A parallelogram, therefore, is a four-sided figure the opposite sides of which are parallel.

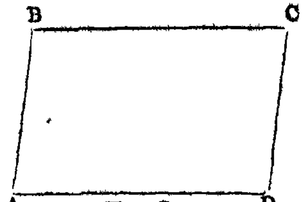


Fig. I.

ABCD, Fig. I, is a parallelogram, for the side AB is parallel to the opposite side CD, and the side BC parallel to the opposite side AD.

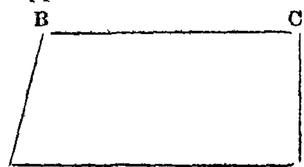


Fig. II.

ABCD, Fig. II, *is not a parallelogram*, for although the sides BC and AD are parallel to one another, the sides AB and DC are *not parallel* to one another.

From Lessons 27 and 28 we see that to construct a square, we require to know the length of only *one* side. If we know the length of one side, we know the length of every side, since they all have the same length. Also we know that every angle is a right angle. Hence a square is completely known, if we know *one* thing, *viz.*, *the length of a side.*

From Lessons 29 and 30, we see, that to construct any required rectangle we must know *two* things, *viz.*, the lengths of *two adjacent sides.* Knowing two adjacent sides, we know the lengths of each side, since opposite sides of a rectangle are equal. Hence, as all the angles of a rectangle are right angles, we can

construct the figure, if we *know the lengths of two adjacent sides.*

But it is evident that we must know *more than the lengths of adjacent sides* in a parallelogram before we can construct it. We must also know the *angle between them.*

Look at the parallelograms in Figs. III and IV.

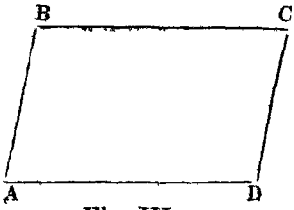


Fig. III.

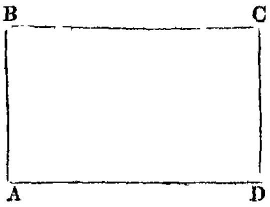


Fig. IV.

The sides AB and AD (Fig. III) have the same lengths as the sides AB and AD (Fig. IV); but the two parallelograms are not therefore equal. They would *not coincide if superposed*, for the angle A (Fig. III) is evidently *less than* the angle A (Fig. IV.)

We must, therefore, know *the size of the angle between the adjacent sides as well as the lengths of these sides.*

Knowing these three things,—the *lengths* of two *adjacent sides* and the *size* of the *angle between them*, we draw the parllelogram as follows:—

(1) From any point we draw straight lines of the given lengths and inclined to each other at the given angle, as AB and AD (Fig. III).

(2) We draw through the end of one of these lines, say AB, (Fig. III) a straight line (BC Fig. III) parallel to the other line (AD Fig. III).

(3) We draw through the end of the other line (AD, Fig. III) a straight line (DC, Fig. III) parallel to the first (AB, Fig. III) to meet the line BC at C.

These four lines AB, AD, BC, and CD, form the parallelogram required, for (1) it is a *parallelogram* since the opposite sides are parallel, (2) it has two adjacent sides of the required lengths, and inclined to each other at the required angle.

EXERCISES.

1. Which of the given four-sided figures are parallelograms?

2 Draw the parallelograms, of which two adjacent sides and the included angles are given below —

No of parallelogram	1	2	3	4	5
Lengths of two adjacent sides, in Cms,	2 and 3	2 and 5	1 and 3	4 and 4	2 7 and 5 4
Included angle	60°	35°	50°	80°	57°

3. Measure the three remaining angles of each of the parallelograms in Ex. 2 Find the sum of each pair of the *adjacent* angles

4. Measure the lengths of the two remaining sides in each of the parallelograms in Ex 2.

5. Measure the lengths of the two diagonals, in each of the parallelograms in Ex. 2 , also the lengths of the two parts into which each diagonal is divided by the other.

6. Verify that a parallelogram is divided by each diagonal into two triangles which are equal in all respects.

7. Form a table, as in Lesson 30, showing in parallel columns the corresponding properties of squares, rectangles and parallelograms.

Lesson 33.

Rhombus.

Looking back to Lessons 27 to 30, we see that squares, and rectangles, are *also parallelograms* for they are four sided-figures, *having their opposite sides parallel*, which is what we mean by a parallelogram. Squares and rectangles are therefore particular forms of parallelograms; they may be called "Special cases."

There is another particular form of parallelogram, besides the rectangle, and square, which we may consider here. It is called the **Rhombus.**

This four-sided figure occupies an intermediate position between the ordinary parallelogram, and the square. It has **all its sides of equal length** like a square, but its **angles are not right angles,** hence it has not at all the erect appearance of a square, but is quite like the ordinary parallelogram. Its special properties we shall learn presently.

Meanwhile let us look at these four kinds of figures, square, rectangle, rhombus and parallelogram, from the point of view of Lesson 32.

(1) To draw a parallelogram we require to know *three* things—two adjacent sides, and the included angle.

(2) To draw a rhombus, we require to know only *two* things—the length of one side, and the angle between two adjacent sides.

(3) To draw a rectangle we also require to know only *two* things—the length of two adjacent sides.

(4) To draw a square, we require to know only *one* thing,—the length of a side.

EXERCISES.

1. Draw the rhombuses, whose sides have the lengths, and are inclined to each other at the angles, given below.

No of Rhombus ..	1	2	3	4
Length of a side (in Cms) ..	5	5	3	3
Angle between adjacent sides .	30°	100°	60°	120°

2. Verify that in a rhombus—
   (1) All the sides have the same length.
   (2) Opposite sides are equal.
   (3) The diagonals bisect each other.
   (4) " " the angles through which they pass.
   (5) " " are prependicular to each other

3. Draw the rhombuses, whose diagonals are 2 and 3, 2 and 4, 5 6 and 2, 7 and 6, 6 and 6 2 Cms.

4. Draw and cut out the rectangles, whose adjacent sides have the same lengths as the diagonals of the rhombuses of Ex. 3, and prove (by cutting each rectangle along both diagonals, and arranging the parts) that each rectangle is twice as large as the corresponding rhombus.

LESSON 34.

Parallel Lines.

Referring to Exs. 6, and 7, of Lesson 31, we see that the farther off the point of intersection of two intersecting straight lines is the *more nearly parallel do the lines become.*

If, for instance, a point N could be taken on the line ABC, Ex. 6, at a distance of, say, 200 Cms. from A, the angle between the lines AN and NO, would be very small, and the lines would seem very nearly parallel. If we considered the exterior angle at O of the triangle ANO, we should find it *very* nearly equal to the interior opposite angle OAN, the difference being the angle at N.

If N were now moved to a distance of 2,000 Cms. from A, the angle at N would be *exceedingly small*, and the exterior angle at O, would be *exceedingly nearly* equal to the interior opposite angle OAN.

If, by any means, we could now make the length of AN very great, say 200,000 Cms., (a little over a mile) it would be exceedingly hard for us,—even with our finest instruments,—to detect the difference between the exterior angle at O, and the angle NAO. The lines would therefore, (by the definition of Lesson 31,) be *practically parallel.*

From this, therefore, we may conclude that two straight lines, about a centimetre apart, may, for all practical purposes, be considered parallel, if their point of meeting is at least, one mile distant from the point, where they are 1 Cm. apart.

They are not really parallel, however, for the exterior angle is not *exactly* equal to the interior opposite angle. The exterior angle is greater than the interior opposite angle, by a *very small angle*, an angle less than the three thousandth part of a degree. Now, although this is a very small angle indeed, and difficult to detect, we can yet detect it. We have

instruments so fine, that we could measure, not only this angle, but angles much smaller than this—even one-tenth of this.

If, however, the lines met at a distance of say 100, or even 50 miles, we can safely say, that *no instrument* yet constructed, *could shew that they were not exactly parallel.*

If, therefore, straight lines about a centimetre apart do not meet within a distance of 50 miles, they may be taken as parallel.

Let us now turn to what is said in books on Geometry on this point:

We find something like the following:—

"Two straight lines in the same plane, (*i. e.*, on the same flat surface), which **do not meet** *however far they may be produced* both ways are said to be parallel."

[The words, "in the same plane" are added because if the two straight lines are not on the same flat surface, they may not be parallel, even though they do not meet. Two pencils, for instance, may be held *at right angles* to each other without meeting, provided they are a little distance apart].

It is also proved that if two straight lines in the same plane do not meet, however far, they may be produced both ways, then, the **alternate angles are equal.**

The definition of Parallel Lines, given in Lesson 31, is, therefore, different from the definition given in books on Geometry, although it can be proved that *the two definitions really agree.*

For if two straight lines are such that any straight line meeting both, *make the alternate angles equal*, we see from Exs. 6 and 7, Lesson 31, that the two straight lines *cannot meet at a finite distance*. They are, therefore, parallel, *by the definition given in books on Geometry*.

Again, if they do not meet at a finite distance, (and are therefore parallel, by the definition given in books on Geometry), *the alternate angles can be proved to be equal.* The lines are, therefere, parallel *by the definition of parallel Lines given in Lesson* 31.

But we may ask, why give *two different* definitions of Parallel Lines.

The reason is this: the definition given in books on Geometry *cannot be applied practically* to determine whether two given lines are parallel or not. To apply it, we must first *produce each of the given lines ever so far both ways*. Now, we find it extremely difficult to draw a *perfectly straight* line, of even 200 Cms., and a line of 200,000 Cms. is for us quite impossible. We, therefore, are *unable to produce a straight line ever so far both ways*, and cannot tell whether two given lines would, or would not meet, if so produced,—*i. e.*, we cannot, by this method, tell whether they are parallel or not. The definition does not furnish us with a *practical test* for the parallelism of lines. It may be quite satisfactory as a mathematical definition—although some doubt even this,—but it is *useless as a practical definition*, for it cannot be applied.

The definition given in Lesson 31, on the other hand, can be applied quite easily. We can in every

case, *readily measure the alternate angles.* If these are equal, the lines are parallel; if not, they are not parallel.

We may sum up what we have learned about parallel lines as follows:—

(1) two lines which are far from being parallel, meet at a short distance;

(2) two lines more nearly parallel, meet at a greater distance;

(3) two lines *very nearly* parallel meet at a *very great* distance;

(4) two lines *exactly parallel* do not meet within any *finite* distance, *however great;* or, as it is sometimes expressed,—*parallel lines meet at an* infinite *distance.*

NOTE—It is *very important* that we should think of parallel lines, in this way, as a *special case* of lines which meet; for, in many branches of Mathematics, it is essential to think of parallel lines as the Limiting Case *of lines whose point of meeting has moved off to an indefinitely great distance.*

REVISION OF CHAPTER IV.

Which of the previous exercises establish the following propositions regarding the four-sided figures considered in this Chapter, *viz.*:—

1. Square.

(1) All the sides have the same length.
(2) Opposite sides are parallel.
(3) All the angles are right angles.
(4) The diagonals are equal in length.
(5) „ „ „ perpendicular to each other.
(6) „ „ bisect each other.
(7) „ „ „ the angles through which they pass.
(8) The ratio,—diagonal to side, is 1·414 approximately.

2. Rhombus.

(1) All the sides have the same length.

(2) Opposite sides are parallel.

(3) „ angles are equal.

(4) The diagonals are perpendicular to each other.

(5) „ „ bisect each other.

(6) „ „ „ the angles through which they pass.

3 Rectangle.

(1) Opposite sides have the same length.

(2) „ „ are parallel.

(3) All the angles are right angles.

(4) The diagonals are equal in length.

(5) „ „ bisect each other.

4 Parallelograms.

(1) Opposite sides have the same length.

(2) „ „ are parallel.

(3) „ angles are equal.

(4) The diagonals bisect each other.

NOTE — The properties of parallelograms are those that are *common* to the first three figures.

CHAPTER V.—AREA.

LESSON 35.

Comparison of Areas.

IN EX. 6, Lesson 32, we saw that a parallelogram is divided by each diagonal, into two parts which fit each other exactly.

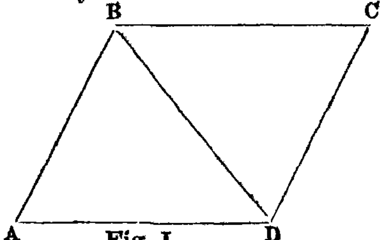


Fig. I.

Thus the diagonal, BD, of the parallelogram, ABCD, (Fig. I) divides the figure into the two equal triangles, ABD and BDC, for if one of these triangles, say ABD, is cut neatly out, and superposed on the other, it will be found to cover the other exactly.

Each of the two triangles is thus *one-half* of the parallelogram, since the two together just make up the parallelogram.

If we had drawn the other diagonal, AC, we should have found, in the same way, that the triangles ACD and ACB, covered each other exactly.

Hence the parallelogram, ABCD, is exactly double of each of the four triangles, ABD, CBD, ACD, and ACB.

Now, what do we mean by saying that the parallelogram is *double* the triangles?

Clearly we mean that it *covers twice as much of the paper*. For, when the triangles, ABD and CBD, (which exactly cover each other) are placed side by side, as in Fig. I, they exactly cover the parallelogram. The amount of the surface of the paper covered by the parallelogram, is, therefore, twice as great as the amount of the surface of the paper covered by the triangle.

We usually express this by saying that the **area** of the parallelogram is *twice* the **area** of each triangle, *i.e.*, we use the single phrase "**area of**" in place of the long phrase, "amount of surface covered by."

The **area** of any closed figure, therefore, is the **amount of surface which it covers**, and this example shows us how one area,—the area of a triangle—may be compared with another area,—the area of a particular parallelogram.

Another example of the comparison of area is given in Ex. 4, Lesson 33 We there compared the area of a rhombus with area of the rectangle, which had its adjacent sides respectively equal to the diagonals of the rhombus.

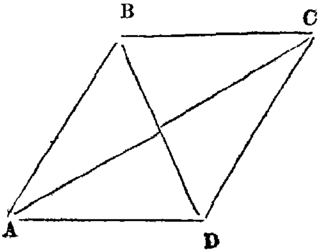


Fig. II.

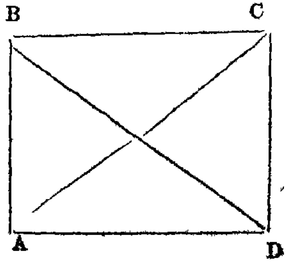


Fig. III.

Suppose the rectangle, ABCD, (Fig. III) has its adjacent sides, AB, and AD, respectively, equal to the diagonals, BD, and AC, of the rhombus, ABCD, (Fig. II.)

If we divide the rectangle into four triangles by cutting it along both diagonals, we find that *each triangle* exactly covers *one-half* of the rhombus.

Thus each of the triangles, ABE, and CDE, (Fig. III) exactly covers *each* of the triangles, ABD and CBD, (Fig. II), and is, therefore, *one-half* of the rhombus.

Similarly, each of the triangles, BEC and AED, (Fig. III) exactly covers each of the triangles ABC and ADC, (Fig. II), and is therefore, one-half of the rhombus.

From this we see—

(1) That the four triangles, into which the rectangle is divided, are of equal area, for each covers one-half of the rhombus.

(2) That each is one-fourth of the rectangle, for the four together make up the rectangle.

(3) That the area of the rectangle is twice the area of the rhombus; for the latter is completely *covered by two of the triangles, while the former requires all four triangles to cover it.*

Areas, like lengths, are therefore compared by *superposition*, although, as we shall see, the comparison of areas is not so easy and simple a process, as the comparison of lengths.

EXERCISES.

1. Draw the parallelograms of which the given triangles are the respective halves. Shew that in each case, three different parallelograms can be drawn.

2. Shew that when the three parallelograms, referred to in Ex. 1, are drawn, the bounding lines form a triangle, whose sides are *twice* the lengths of the sides of the original triangle. Show that its area is *four* times the area of the original triangle.

3. Shew how to cut the triangle, ACD, (Fig, I, Lesson 35) into two parts, which, together, exactly cover the triangle ABD, (same Fig.)

4. Shew how to cut the triangle, AEB, (Fig, III, Lesson 35) into two parts, which, together, exactly cover the triangle, BEC, (same Fig).

5. Prove by joining the middle points of opposite sides of a square, whose side is 2 Cms., that the area of such a square is four times the area of a square whose side is 1 Cm

6. Prove, by joining the middle points of opposite sides of a square whose side is 4 Cms., that the area of such a square is four times the area of a square whose side is 2 Cms.

7. How many squares of 1 Cm side are required to cover (1) a square of 2 Cms. side, (2) a square of 4 Cms side?

8. How could you shew, that a square of 3 Cms. side covers nine times the surface of a square of 1 Cm. side?

9. Prove that if the length of the side of one square is twice the length of the side of another square, the area of the former square is *four times* the area of the latter.

Lesson 36.

Unit of Area.

In Lesson 10, we saw that the lengths of lines are best compared by finding how often each contains a standard line—the centimetre. If one line contains 5·62 Cms., while another contains 8·43 Cms., we know that the latter line is longer than the other by a line half the length of that other; the one is half as long again as the other, or 1·5 times the other.

If their lengths were 4·73 and 3·25 Cms., respectively, we know that the one is 4·73÷3·25 or 1·4054 times the other. And so for other lengths.

In the same way, the *area of figures* can be best compared by finding how often each figure contains a particular *standard area*, which may be called the **unit of area.**

The process of comparison, is however, not so easy as in the case of straight lines. For in measuring straight lines, we merely *superposed* our Cm. Scale on the line to be measured, whereas in comparing areas, *mere superposition is seldom sufficient.*

For example, the two triangles, ABD, ACD, (Fig. I, Lesson 35) have shapes so very different, that mere superposition of the one on the other gives us very little information regarding their relative areas. We could not discover that their areas are equal by merely superposing the one on the other.

In such cases it is *not enough to superpose*; we must *alter the shape*, as well. We must for example, cut the triangle, ACD, into two parts, and readjust them to form a triangle, of the *same shape* as the triangle, ABD. Having made both triangles of the *same shape*, we can now superpose them, and thus learn that they are equal in area. Had it been impossible to make them both of the same shape, we could not have compared their areas.

In comparing the areas of figures, *difference of shape* is often a very serious obstacle. There are a large number of cases, where this difficulty is very

great—indeed so great as to be almost insurmountable. In other cases, the difficulty is not so formidable.

A few such cases we shall now consider.

The figure whose area is most easily calculated is the square.

This is because all squares have the *same shape*; in comparing the areas of different squares, difficulties arising from difference of shape, do not appear.

For this reason, it has been found most convenient to take, as unit of area, *the area of a particular square.* The unit of area that has been selected, is the area of a square, whose side is one centimetre. This unit of area is called a "square centimetre,"— usually written "Sq. Cm."

We are said to *measure the area* of any given figure, when we determine the number of units of area or Sq. Cms., that are required to *cover the surface of the figure.*

This number is said to be the measure of the area, or, for shortness, the area of the figure.

The area of a figure, therefore, is the number of Sq. Cms. that completely cover the figure. (This number *may be fractional.*)

We saw, in Ex. 5 of last lesson, that the number of Sq. Cms. required to cover a square of 2 Cms. of side, is 4.

Therefore the area of a square 2 Cms. side is 4. Similary in Ex. 6 we saw that a square of 4 Cms. side was completely covered by *four* squares each of

2 Cms. side ; hence the area of a square of 4 Cms. side is 16.

In Ex. 9, we saw that if we know the area of *any square,* the area of the square, whose sides are *twice as long,* can always be found,—the **area** of the latter square is **four times** the area of the former. Hence we know the areas of squares, whose sides are 2, 4, 8, 16, 32, &c., Cms. respectively.

EXERCISES.

1. What area has been selected as the *unit of area* ? What reason has been given for the selection of this unit ?

2. What is the meaning of the phrase, "measure of an area" , what equivalent expression is sometimes used ?

3. What is the area of a square of 16 Cms. side ?

LESSON 37.

Area of Squares.

Let us now consider how to find the area of *any given square.* How can we find how many Sq. Cms. will exactly cover it ?

1. Suppose it has an *exact* number of Cms. in each side, say, five.

We first divide *each* side into centimetre lengths by marks 1 Cm. apart. Then we draw a series of straight lines from the marks on one side, to the corresponding marks on the opposite side. We, thus, divide the square into five rectangular strips, each 1 Cm. broad, and 5 Cms. long. We now draw a similar series of straight lines to join the marks on the other pair of opposite sides, and find that we have now divided *each* strip into 5 *equal parts,* and that *each part is a Sq. Cm.*

Since, therefore, there are five strips, each containing 5 Sq. Cms, the total number of Sq. Cms. required to cover the square of 5 Cms. side, is 5×5, or 25.

Fig. I shows how a square of 5 Cms. side may be divided into 25 Sq. Cms. in the way described.

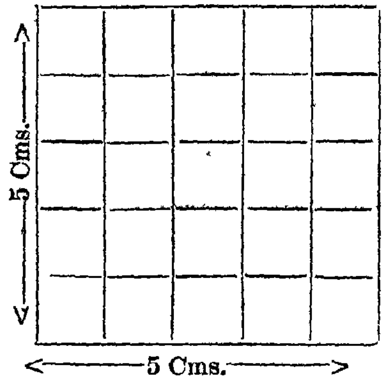


Fig. I.

Similarly, if the side of a square contained 7 Cms, we could, as above, divide the square into 7 strips, *each* containing 7 Sq. Cms; therefore the area of the square is 7×7, or, 49 Sq. Cms.

In a similar way, the area of *any square*, whose side is an exact number of Cms., may be found.

2. Suppose the side of square contains an *exact* number of *Cms.*+an *exact* number of *millimetres*; suppose its length is 5 Cms.+3 Mms., or 5·3 Cms.

(1) Consider how many squares, of 1 Mm. side, exactly cover 1 Sq. Cm. It is easy to see, that, by dividing each side of the Sq. Cm. into millimetres and

joining corresponding points (as in Fig. I), we divide the Sq. Cm. into ten strips, each containing ten little squares of 1 Mm. side. Hence 10×10, or, 100 little squares (called square millimetres—written "Sq. Mm.") exactly cover 1 Sq. Cm.

(2) Now, in our square, each side contains 5·3 Cms. *i.e.* 53 Mms; as before we may divide it into 53 strips, each, containing 53 Sq. Mms; therefore the total number of Sq. Mms. in this square is 53×53 or 2809.

(3) But as each Sq. Cm. contains 100 Sq. Mms. and 1 Sq. Mm., therefore, contains $\frac{1}{100}$ Sq. Cm., it is evident that 28·09 Sq. Mm. contains $\frac{2809}{100}$, or, 28·09 Sq. Cms. Now $2809 = 5·30 \times 5·3$.

Hence in this case, also, the **area of the square is the number obtained by multiplying the number** (whole, or fractional) of **Cms. in each side, by itself.**

3. Suppose the length of the side is not exactly 5·3, but is 5·37 Cms. How shall we find its area?

(1) As before, we see, that each Sq. Mm. contains 100 minute squares, whose sides are one-tenth of a Mm., therefore 1 Sq. Cm. contains 100×100 or 10,000 such squares.

(2) The side of the given square contains 537 tenths of a Mm., and the square could, therefore, as before, be divided into 537 rectangular strips each containing 537 minute squares of one tenth of a Mm. side. Hence, in all, the given square contains 537×537, or, 288369 such squares.

(3) Since 10,000 such squares make up 1 Sq. Cm.,
and therefore 1 such square $= \frac{1}{10000}$,, ,,
∴ 288369 such squares $= \frac{288369}{10000}$,, ,,
$= 28{\cdot}8369$,, ,,
$= 5{\cdot}37 \times 5{\cdot}37$,, ,,

Hence the rule holds in this case also.

EXERCISES.

1 Write down the rule for finding the area of any square

2. Find the area of all the squares in the Exercises of Lessons 27 and 28

Lesson 38.

Area of Rectangles

Although rectangles may have a great variety of shapes, it is yet quite easy to determine their areas. All their angles are right angles; hence any rectangle can be covered by small squares, and its area thereby determined.

1. Consider a rectangle, whose sides contain an *exact* number of Cms.; let us suppose, they contain 2 and 3 Cms., respectively. By dividing each side, as before, into Cms. lengths, and joining corresponding marks on opposite sides (see Fig. I), we divide the figure into two strips, each containing 3 Sq. Cms.

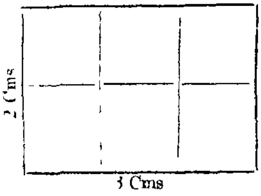


Fig. I.

The rectangle would, therefore, be exactly covered by 2 × 3, or 6 Sq. Cms.

Similarly, if the lengths of the sides had been 7, and 11 Cms. respectively, we could have divided the figure into 7 strips, each containing 11 Sq. Cms., and the area would have been 7 × 11, or 77 Sq. Cms.

2. If the sides are 2·4 and 3·1 Cms. respectively, we may shew, as in Lesson 37, that the figure is exactly covered by 24 × 31, or 744 Sq. Mms., *i. e.* by 7·44 Sq. Cms.

3. If the lengths were 2·43 and 3·17 Cms. respectively, we could similarly shew, that the surface would be exactly covered by 243 × 317, or, 77031 squares of $\frac{1}{10}$ Mm. side.

But as 10,000 of these small squares are required to cover a Sq. Cm. we see that the area is $\frac{77031}{10000}$ or 7·7031 Sq. Cms.

Now, as 7·7031 = 2·43 × 3·17, we see that in every case the *area of a rectangle* is the number found by multiplying together number of Cms. (whether *whole or fractional*) in the adjacent sides.

EXERCISES.

1. State the rules for finding the areas of rectangles, and squares, and shew that the rule for rectangles applies also to squares.

2. Find the areas of the rectangle in the Exercises of Lessons 29 and 30.

3. The areas of seven rectangles, and the lengths of one side of each, are given in the following table. Draw the rectangles.

No of rectangle	1	2	3	4	5	6	7
Area of rectangle in Sq Cms	14	24	10 8	7 60	8 1	16 35	18 5274
Length of one side of the rectangle	2	4	3	1 9	3	7 5	4 23

4. Draw the squares, that have the same areas as the rectangles of Ex. 3 (above).

(Ex. 4 may be omitted by those unacquainted with *Square Root.*)

Lesson 39.

Area of Parallelograms.

In determining the number of Sq. Cms. required to cover a square, or a rectangle, we experienced no difficulty from variety in shape.

In the case of parallelograms, however, this is not so. The angles of parallelograms are *not* right angles; our first step, therefore, in determining the area of a parallelogram, is to *alter the shape* of the figure, by cutting off parts and readjusting them, so as to form either a *rectangle* or a *square*. Having changed the parallelogram to a rectangle, or square of the *same area*, we determine the area of the parallelogram by measuring the area of the equivalent rectangle or square, as in Lessons 37 and 38.

How then can a parallelogram be changed to a rectangle without altering its area?

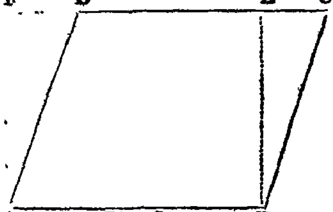


Fig. I.

Consider the parallelogram, ABCD, (Fig. I) If we cut it along the dotted line DE, (which is perpendicular to BC) we shall find that the part, DEC, so cut off, can be adjusted to the other end, AB, of the parallelogram, so as just to cover the part marked off by the dotted lines, AF and FB, and that it will in this position form with the rest of the original parallelogram, the figure ADEF, which will be found to be a rectangle.

Now the area of this rectangle is (Lesson 35) found by multiplying the number of Cms, in the line, AD, by the number in DE.

Hence the area of the parallelogram, ABCD, being equal to the area of the rectangle, ADEF, is obtained *by multiplying* **the length of one side (AD) by the length of the perpendicular distance (DE) between that side and the opposite side of the parallelogram.**

The perpendicular distance between opposite sides is sometimes called the **altitude.**

EXERCISES.

1. Measure the areas of the parallelograms in Ex. 2, Lesson 82.

2. Measure the areas of the rhombuses in Ex. 3, Lesson 33, and compare the results with the areas of the rectangles in Ex. 4, of the same Lesson.

3. Verify from measurements of the parallelograms in Ex. 2, Lesson 32, that in a parallelogram the product of the number of Cms. in the perpendicular distance between opposite sides into the number of Cms. in one of these sides, is the same for *each* pair of opposite sides.

4. The parallelograms, of which one side and an adjacent angle are given in the following table, have each the same area, *viz.* 25·75 Sq. Cms.; draw the parallelograms.

No of parallelogram	1	2	3	4	5
Length of one side	4·5	5	5 15	3 1	8 3
Adjacent angle	45°	50°	90°	65°	30°

5. Each of five parallelograms, having each an area of 25 75 Sq. Cms. have one side, 8 3 Cms. in length, and adjacent sides of the lengths given in Ex. 4; find the angles of each parallelogram. (First draw the parallelograms, then measure the angles.)

6. If the sides, of length 8·3 Cms., in the parallelograms of Ex. 5, are drawn so as to be in one straight line, verify (1) that the opposite sides all lie in another straight line; (2) that the first and second straight lines are parallel.

7. If the parallelogram, ABCD, (Fig. I,—Lesson 39,) were such that the foot of the perpendicular from D, on the opposite side, fell not on BC but on BC, *produced*, show how the parallelogram must be cut so as to form a rectangle having AD as one side.

8. Draw squares of the same area as the given parallelograms.

(Ex. 8 may be omitted by those unacquainted with *Square Root*).

Lesson 40.

Area of (1) Triangles, (2) Any figure bounded by Straight Lines.

In finding the area of a triangle, we are again met by a difficulty due to *shape* and our first step must be to *change the shape* to some one or other of the figures, whose areas we have already found.

We shall find that the *parallelogram* is the most suitable shape to change a triangle into, for we saw in Lesson 35, that every parallelogram is made up of two equal triangles each having an area equal to half the area of the parallelograms.

If, therefore, we can find the parallelogram, of which a given triangle is the half, we can find the area of the given triangle. The problem, therefore, is **to draw a parallelogram which has twice the area of the given triangle.**

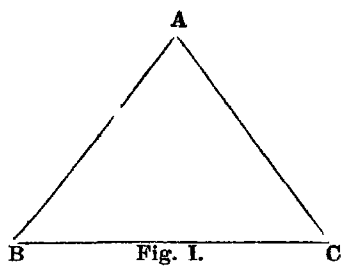


Fig. I.

Suppose ABC, (Fig. I) is the given triangle. How can we apply to ABC another equal triangle, so that the two shall together form a parallelogram?

This may be done in three ways (see Ex. 1, Lesson 35).

(1) We may, as in Fig. II, apply the triangle ACD, (which is equal in all respects to the triangle, ABC,) to the side, AC, of the triangle, ABC, so that the side AD, of the triangle, ADC, is opposite to the equal side, BC, of the triangle, ABC.

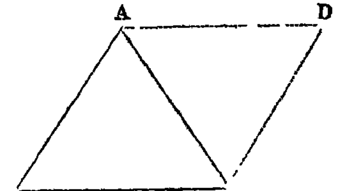


Fig. II.

Since the angle, ACB, is equal to the angle, CAD, the side, AD, is parallel to the side BC.

Similarly, the side, DC, is parallel to the side AB. The figure, ABCD, has therefore its opposite sides parallel, and is, therefore, a parallelogram.

Now the area of this parallelogram

=BC × (perpendicular from C on AD),

=BC × (perpendicular from A on BC).

Hence the area of the triangle

=$\frac{1}{2}$BC × (perpendicular from A on BC).

(2) We could, as in Fig. III, apply the equal triangle to the side, AB,

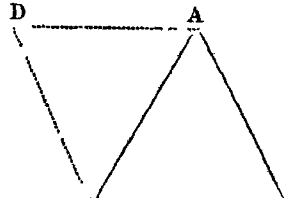


Fig. III.

and obtain the parallelogram, ACBD, (Fig. III); or—

(3) We could apply the equal triangle to the side BC, and obtain the parallelogram, ABDC, (Fig. IV).

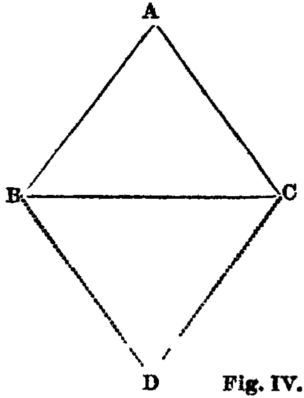


Fig. IV.

All these parallelograms, have the same area, since each has twice the area of the triangle ABC.

From an examination of the above figures (II, III, or IV) we see that the area of the parallelograms may be denoted either by BC × (perpendicular from A on BC),

or by CA × („ „ B „ CA),

or by AB × („ „ C „ AB).

Hence we see, that the **area of any triangle** is **found by multiplying half the length of any side by the length of the perpendicular on it from the opposite angle.**

The perpendicular from one angle of a triangle on the opposite side is sometimes called the **altitude** of the triangle, and the opposite side the **base.**

The area of a triangle may, thus, be given as

$=\frac{1}{2}$ base × **altitude.**

But it must not be forgotten that this statement has *no meaning by itself*; it is impossible for us to multiply one *line* by another *line*. Lines cannot be mutiplied together. Only *numbers* can be multiplied together.

We must carefully bear in mind, therefore, that the statement, "Area of a triangle=$\frac{1}{2}$ base × altitude" is merely a shortened form of the following:—

To find the *number* of Sq. Cms., which would just cover a given triangle, multiply *half the number of Cms.* in the *base* of the triangle, by the *number of Cms.* in its *altitude*.

From the above we can find the area of *any* figure which is bounded by *straight lines*, whatever its shape or the number of its sides may be. For we can divide it into a number of *triangles* the area of each of which we can determine by the method of this Lesson. The area of the figure is the sum of the areas of the various triangles.

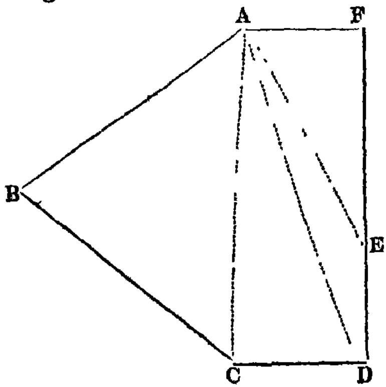


Fig. V.

Thus the area of the irregular six sided figure, bounded by the straight lines, AB, BC, CD, DE, and EF, (Fig. V) can be found by joining *any* angular point, (say A) to the *other* angular points, C, D, and E, and thus dividing the figure into four triangles, the area of each of which can be found.

EXERCISES.

1. Find the areas of the triangles in Ex. 1, Lesson 24.

2 Verify, from measurements of these triangles, that the product of the length of a side of a triangle, into the length of the perpendicular on that side from the opposite angle is the same, *whichever side of the triangle is taken.*

3. The area of each of the four triangles, two of whose sides are given in the following table, is 10 Sq. Cms.; draw the triangles.

No of triangle	1	2	3	4
Length of sides	4 and 5	6 and 4	6 4 and 3 5	7 3 and 4 5

4. Find the areas of the given figures.

5 What do you mean by the following statements :—

(1) The area of a surface is 75 84

(2) The area of a square = $(\text{side})^2$

(3) „ „ rectangle = product of adjacent sides.

(4) „ „ parallelogram = base × altitude

(5) „ „ triangle = $\frac{1}{2}$ base × altitude.

END

APPARATUS.

The apparatus required for this Course may be obtained from the "Scientific Apparatus Manufacturing Co," Colonelganj, Allahabad, at the prices (exclusive of carriage) noted below —

Name or description of apparatus.	Each.			Per dozen (down to ½ dozen)			Per hundred (down to 25)		
	Rs	a	p	Rs	a	p	Rs	a	p
SCALE, 50 centimetres, divided to millimetres (paper)	0	1	0		.		6	0	0
Do do do (mounted on wood)	0	3	0	2	2	0	18	0	0
Do Wrought Iron, plain, 25 Cm. (to Mm)	2	8	0	40	8	0			
Do do do 50 do	6	8	0						
Do do do 100 do	13	0	0						
Do do do 25 do bevelled edge, scale up to edge, may be used as straight edge	4	8	0	46	2	0			
STRAIGHT EDGE, W Iron, thick, with bevelled edge	1	8	0	17	0	0			
Do do do thinner, do	0	12	0	8	4	0			
Do do do plain	0	8	0	5	4	0			
Do do do wood	0	4	0	2	10	0			
PROTRACTORS, paper	0	2	0	1	6	0	12	0	0
Do do mounted on wood	0	4	0	2	12	0	24	0	0
Do do brass	0	13	0	9	0	0			
Do do Ebonite	1	2	0						
COMPASSES, brass steel pointed	0	11	0	7	8	0			
Do W. Iron, rough	0	8	0						